IMPROVING THE MAXIMUM TRANSMISSION DISTANCE IN CV-QKD USING A NOISELESS LINEAR AMPLIFIER

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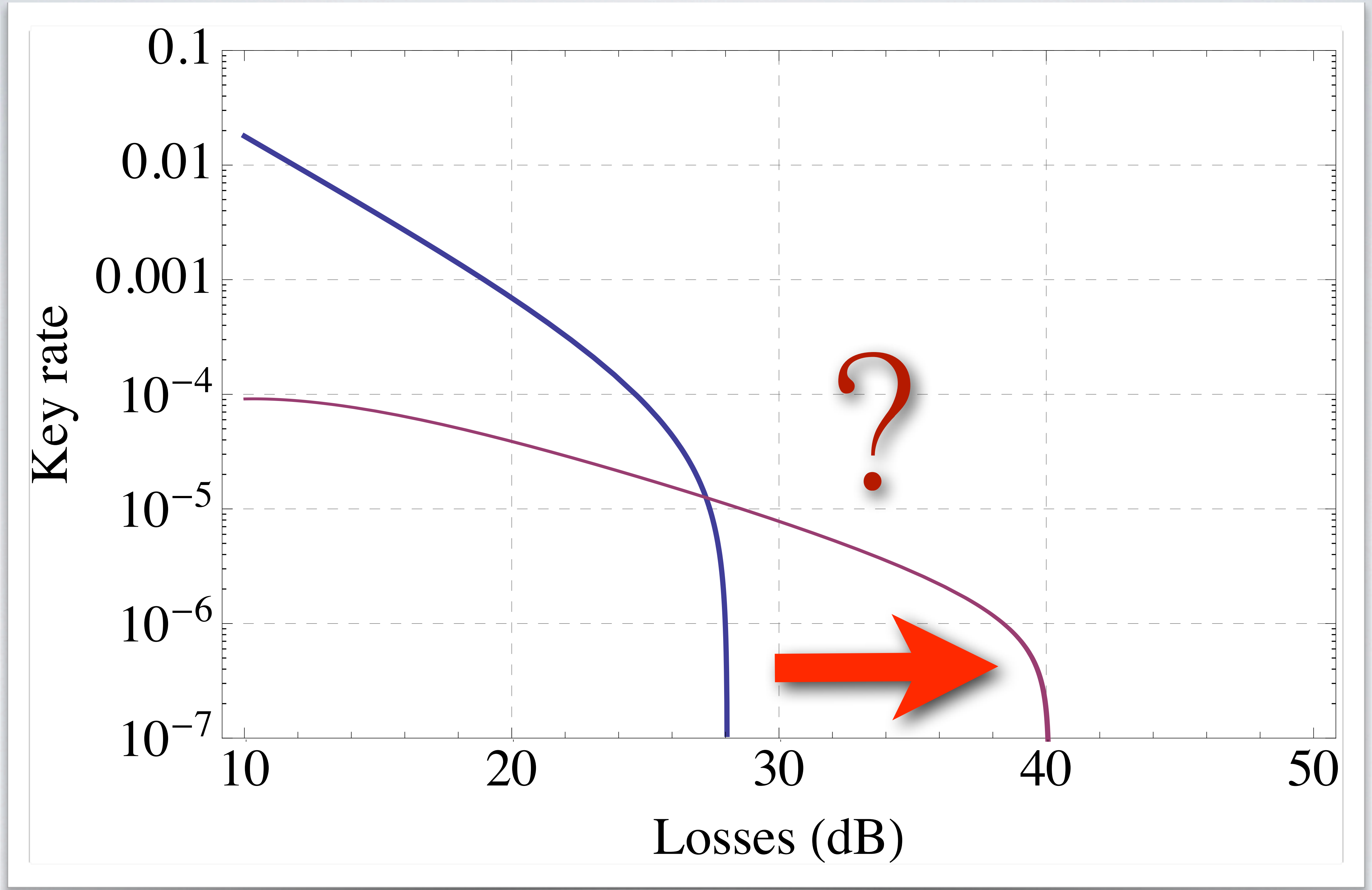
QCRYPT 2012



Can we increase this maximum distance ?

OUTLINE

I. Continuous-variable & coherent states QKD

II. Heralded Noiseless Linear Amplifier (NLA)

III. Improvement of CV-QKD performances with the NLA

I.Continuous-variable & coherent states QKD

Discrete variables

- Decomposition on a discrete basis $|\psi\rangle = \sum_n c_n |n\rangle$

Continuous variables

- Decomposition on a continuous basis $|\psi\rangle = \int \mathrm{d}x\ \psi(x)|x\rangle$

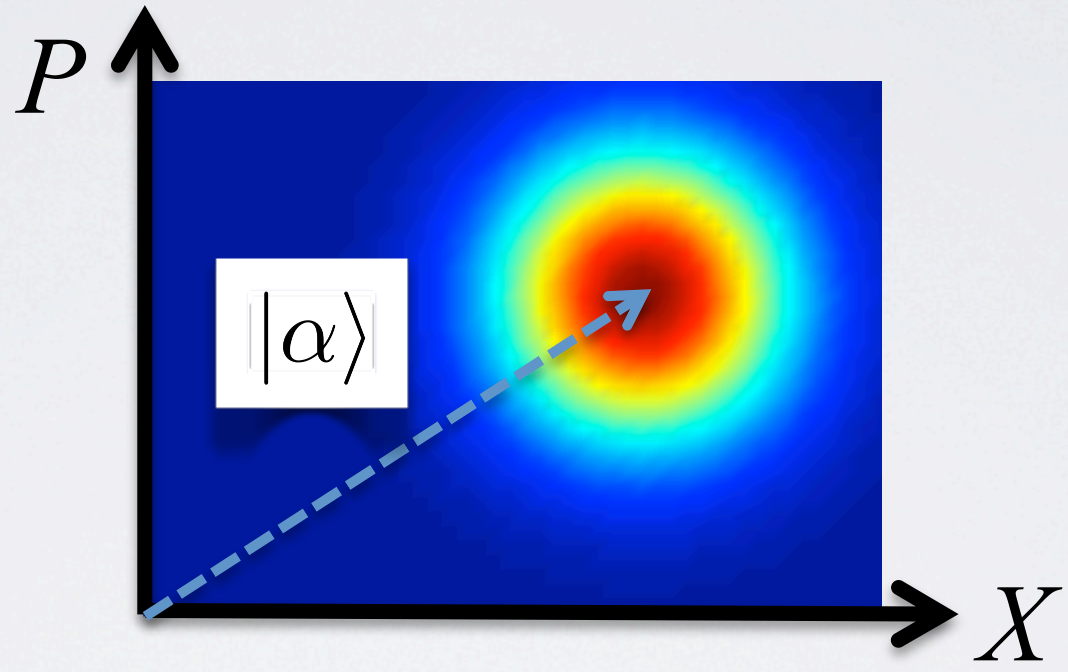
- Quadrature operators $\hat{\boldsymbol{X}}$ and $\hat{\boldsymbol{P}}$ = projection of the field's amplitude in the phase space, similar to the position and momentum for a massive particle

$$\hat{\boldsymbol{X}} = \left(\hat{\boldsymbol{a}} + \hat{\boldsymbol{a}}^\dagger\right)\sqrt{N_0}$$

$$\hat{\boldsymbol{P}} = \left(\hat{\boldsymbol{a}}^\dagger - \hat{\boldsymbol{a}}\right) i\sqrt{N_0}$$

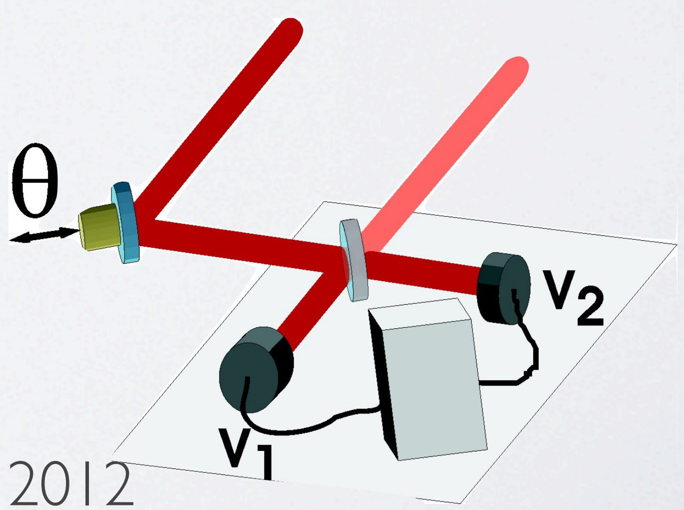
Wigner function

- Quasiprobability distribution



Quadrature measurement

- Homodyne detection



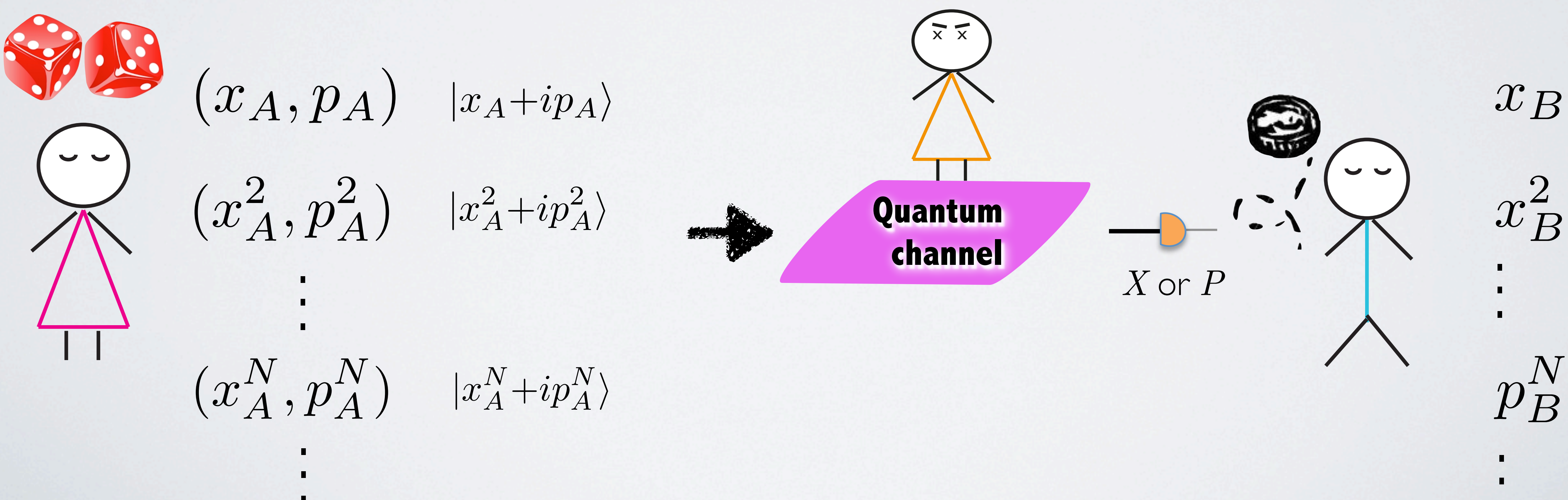
GG02 PROTOCOL

PRL **88**, 057902 (2002)

Nature **421**, 238 (2003)

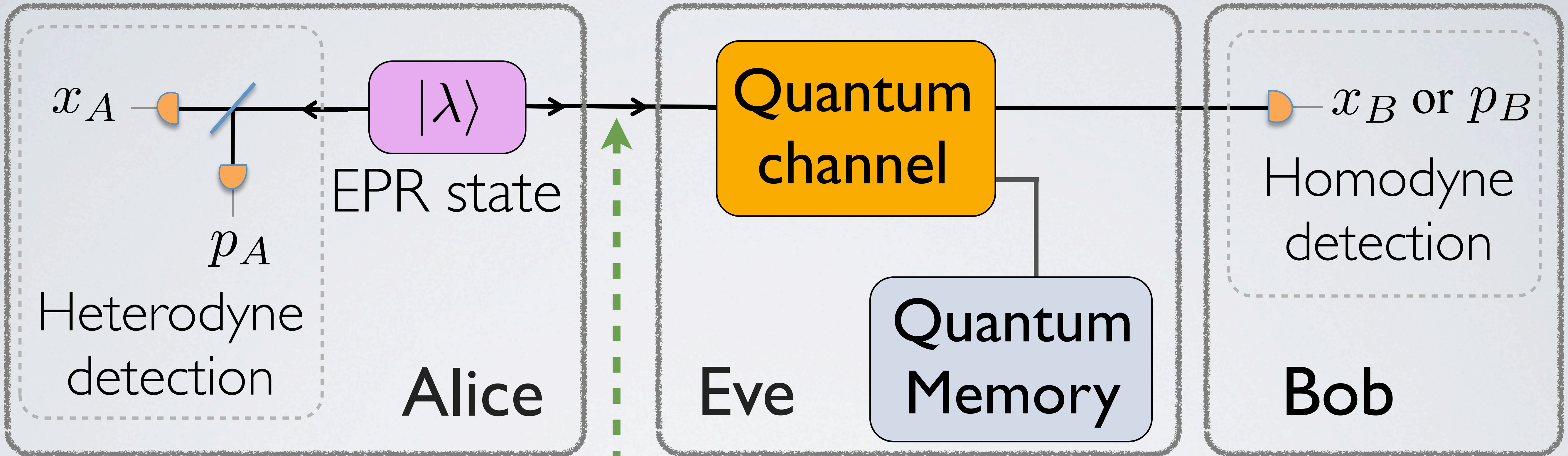
Quantum part

- Alice randomly selects x_A and p_A from a Gaussian distribution of variance V_A
- The state $|x_A + i p_A\rangle$ is sent to Bob
- Bob randomly measures the X or P quadrature



GG02 PROTOCOL

Equivalent Entanglement-Based version



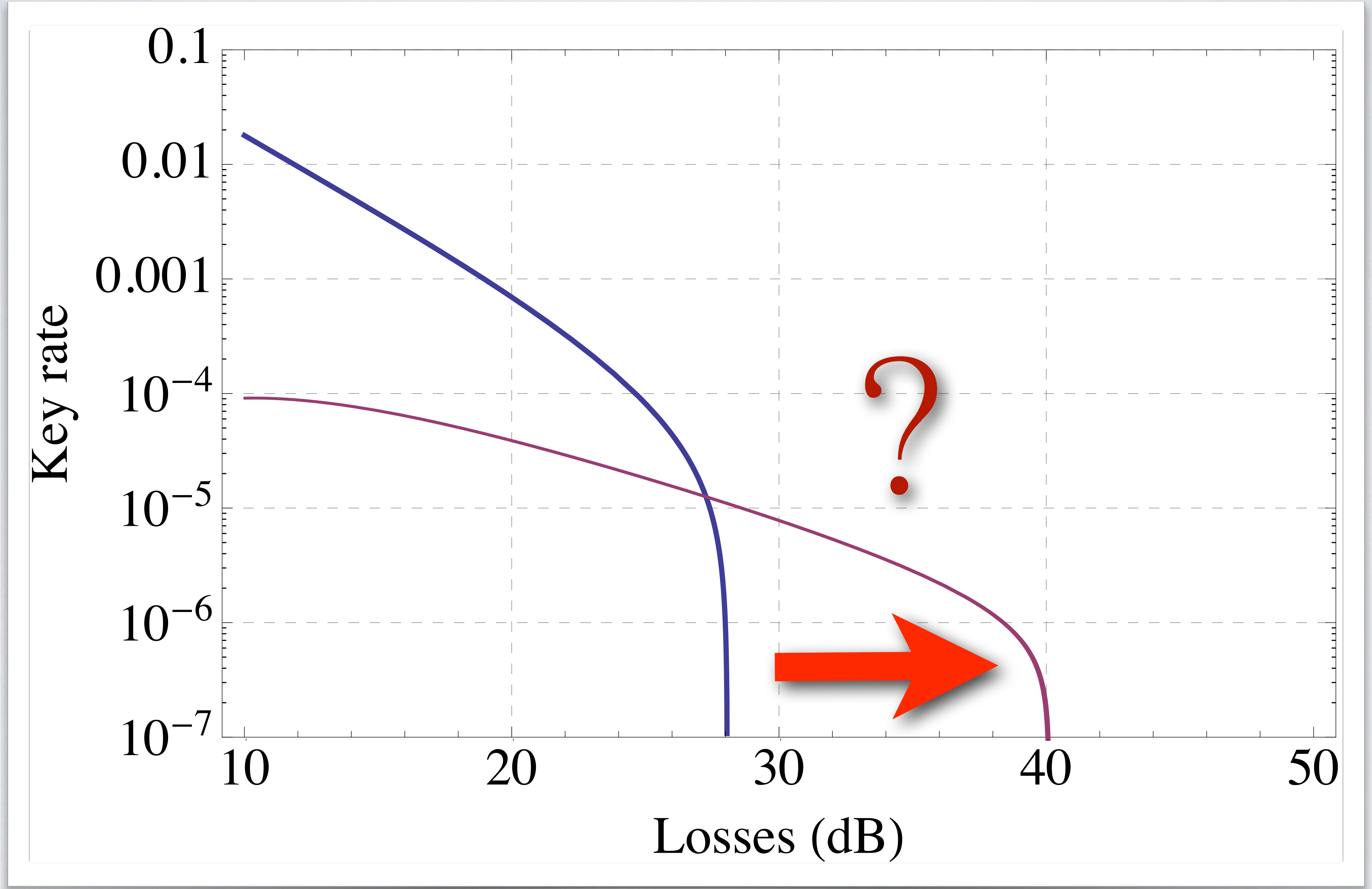
Source of coherent states with:

- amplitude proportional to $\lambda(x_A + ip_A)$
- variance modulation $V_A = \frac{1+\lambda^2}{1-\lambda^2} - 1$

Quantum Info. Comput. **3**, 535–552 (2003)

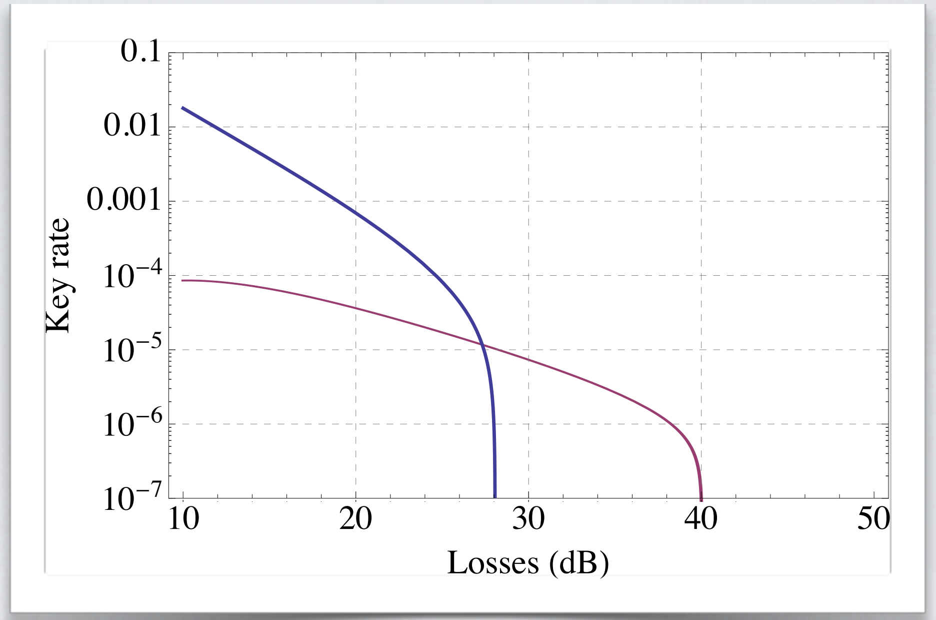
RMP **84**, 621 (2012)

LIMITS

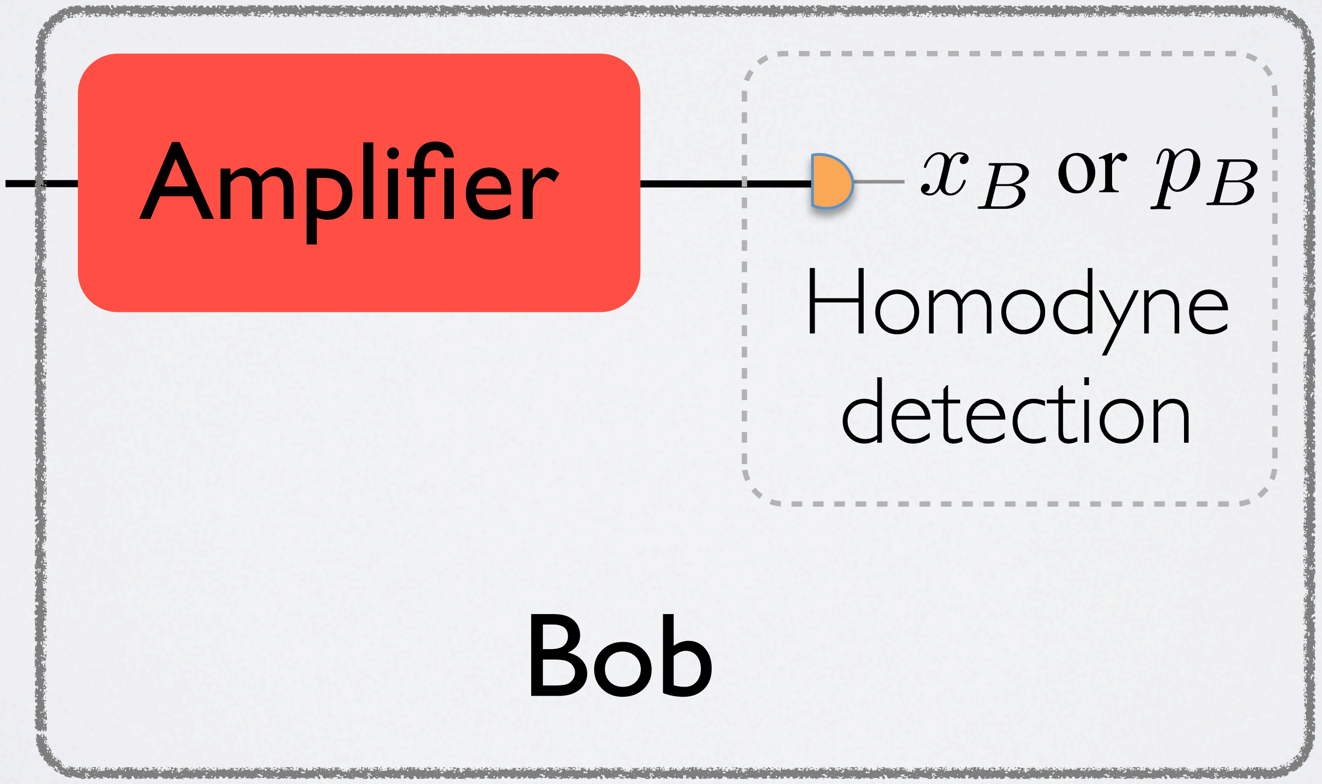


Can we increase this maximum distance ?

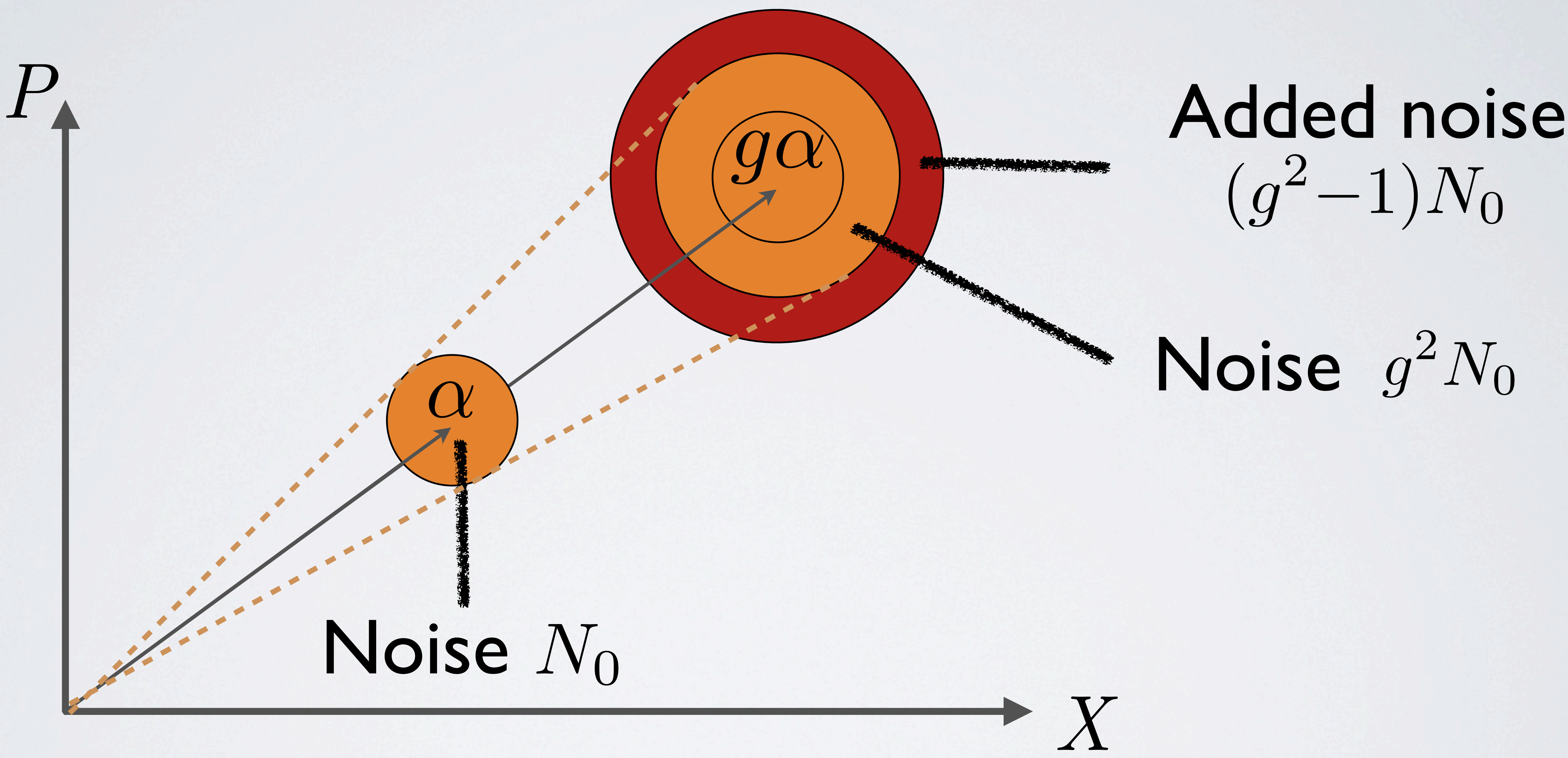
LIMITS



Maybe with an amplifier in Bob's station ?



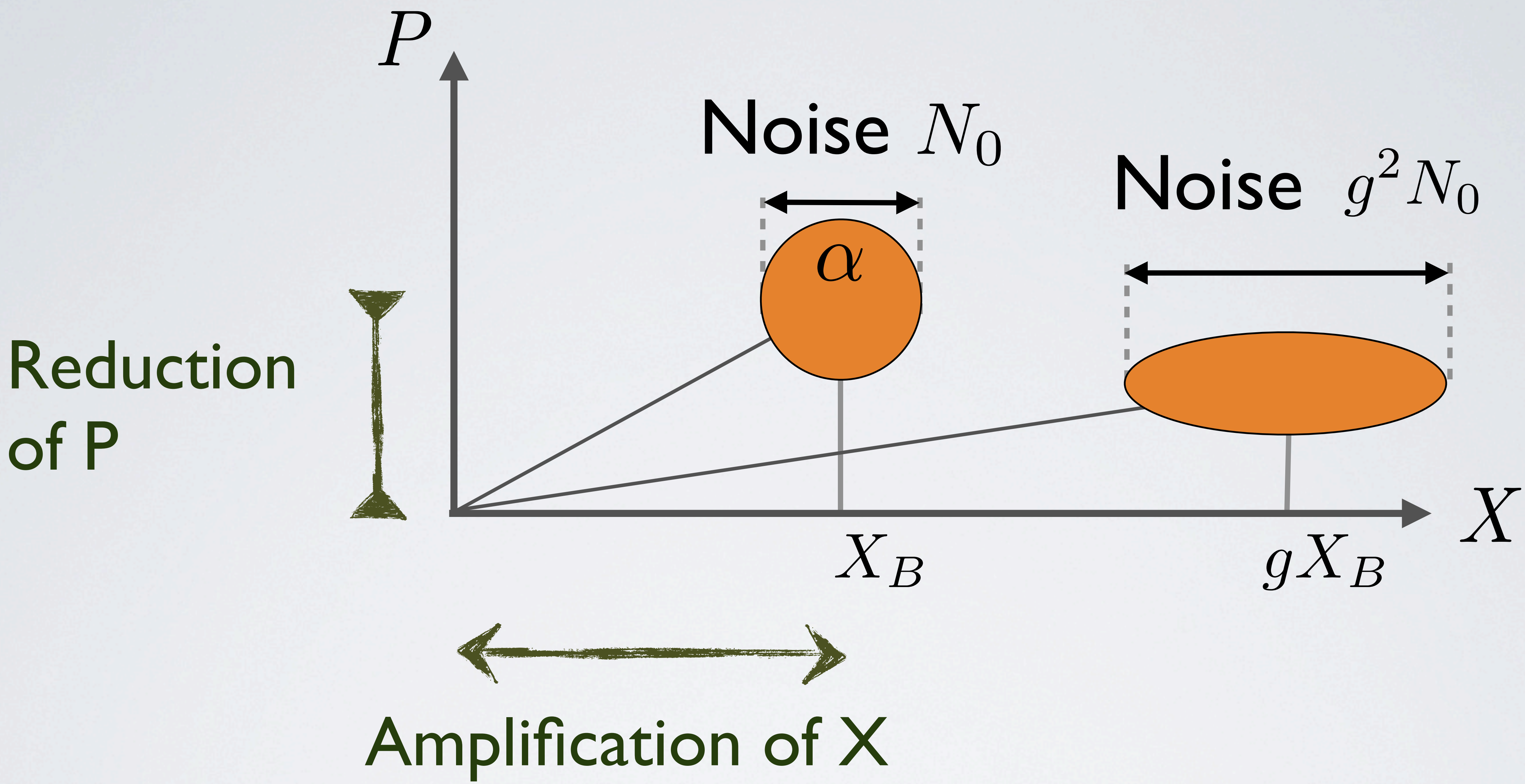
DETERMINISTIC PHASE INSENSITIVE AMPLIFIER



- Must add extra noise

Phys. Rev. D **26**, 1817 (1982)

DETERMINISTIC PHASE SENSITIVE AMPLIFIER



- Doesn't add extra noise → preserves the SNR
- Still amplifies the initial noise
- Only compensates homodyne imperfections

Journal of Physics B **42**, 114014 (2009)

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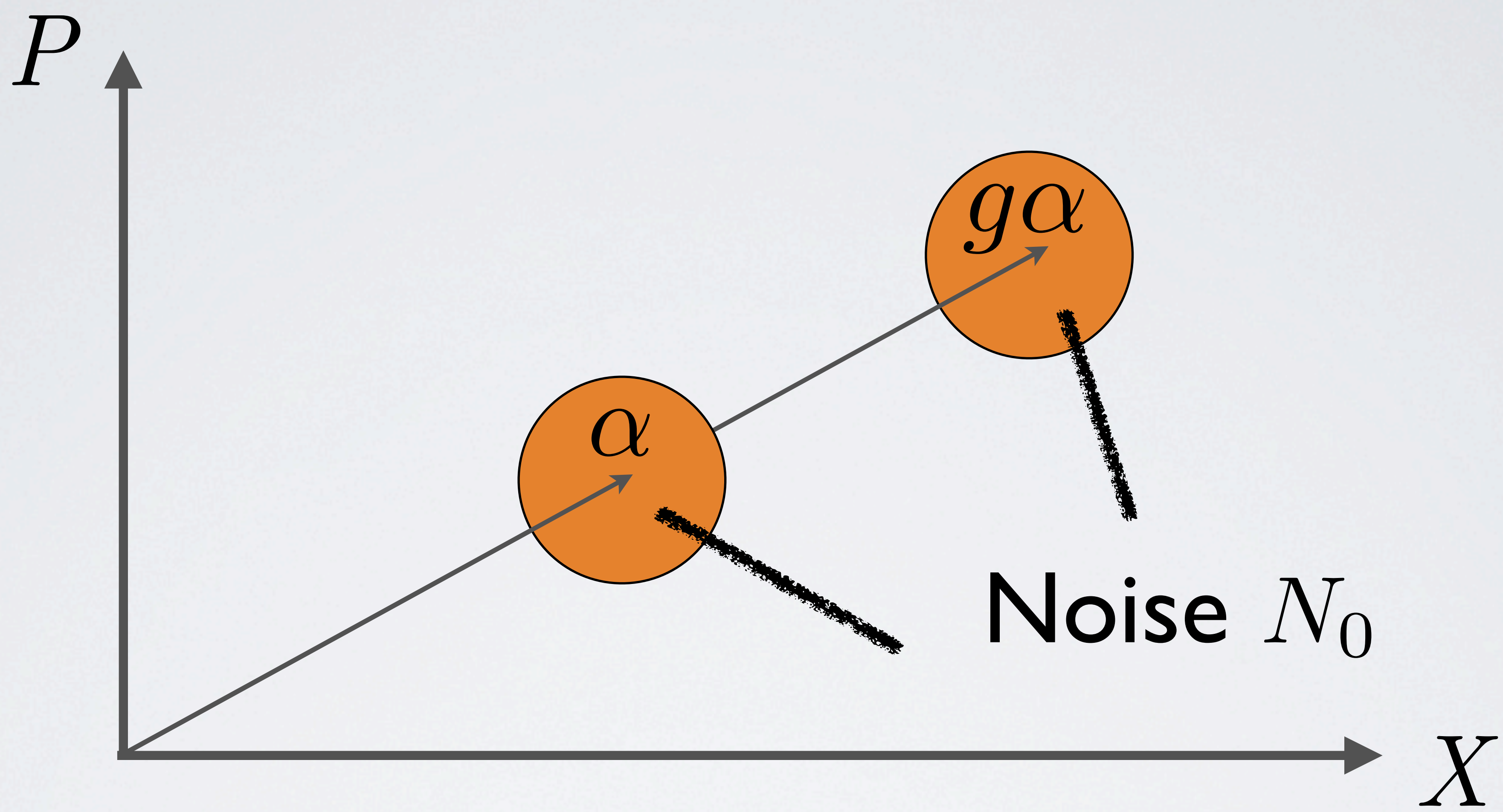
What happens if the amplifier is allowed to be non deterministic?

II. Heralded Noiseless Linear Amplifier (NLA)

THE NOISELESS AMPLIFIER



- Performs the transformation $|\alpha\rangle \to |g\alpha\rangle$
- Phase insensitive, but doesn't add extra noise
- Doesn't amplify the input noise

T.C.Ralph and A.P.Lund, arXiv:0809.0326 (2008)

THE NOISELESS AMPLIFIER

Description of the NLA

- Cannot be unitary ➡ must be probabilistic
- Described by an unbounded operator $g^{\hat{n}}$ $(g^{\hat{n}}|n\rangle = g^{n}|n\rangle)$

Transformation of usual states ➡ Gaussian operation

- Coherent state

$$g^{\hat{n}}|\alpha\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} g^n |n\rangle \propto |g\alpha\rangle$$

- EPR state

$$g^{\hat{n}}|\lambda\rangle = \sqrt{1-\lambda^2} \sum_{n=0}^{\infty} \lambda^n g^n |n,n\rangle \propto |g\lambda\rangle$$

- Thermal state

$$g^{\hat{n}} \hat{\boldsymbol{\rho}}_{\mathrm{th}}(\lambda) g^{\hat{n}} = (1-\lambda^2) \sum_{n=0}^{\infty} g^{2n} \lambda^{2n} |n\rangle\langle n| \propto \hat{\boldsymbol{\rho}}_{\mathrm{th}}(g\lambda)$$

THE NOISELESS AMPLIFIER

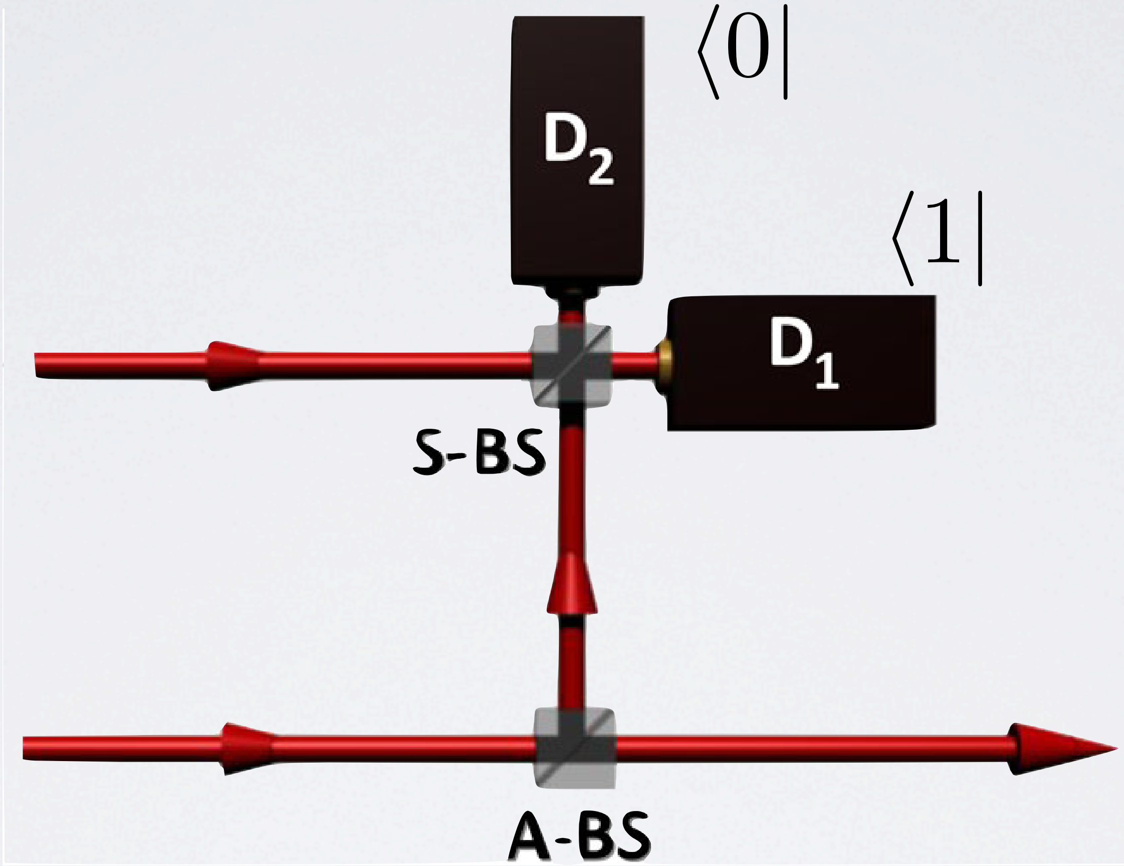
Experimental implementation (quantum scissors)

arXiv:0809.0326 (2008)

PRL **104**, 123603 (2010)

$|\alpha\rangle \simeq |0\rangle + \alpha|1\rangle$

$|1\rangle$



$|0\rangle + g\alpha|1\rangle$

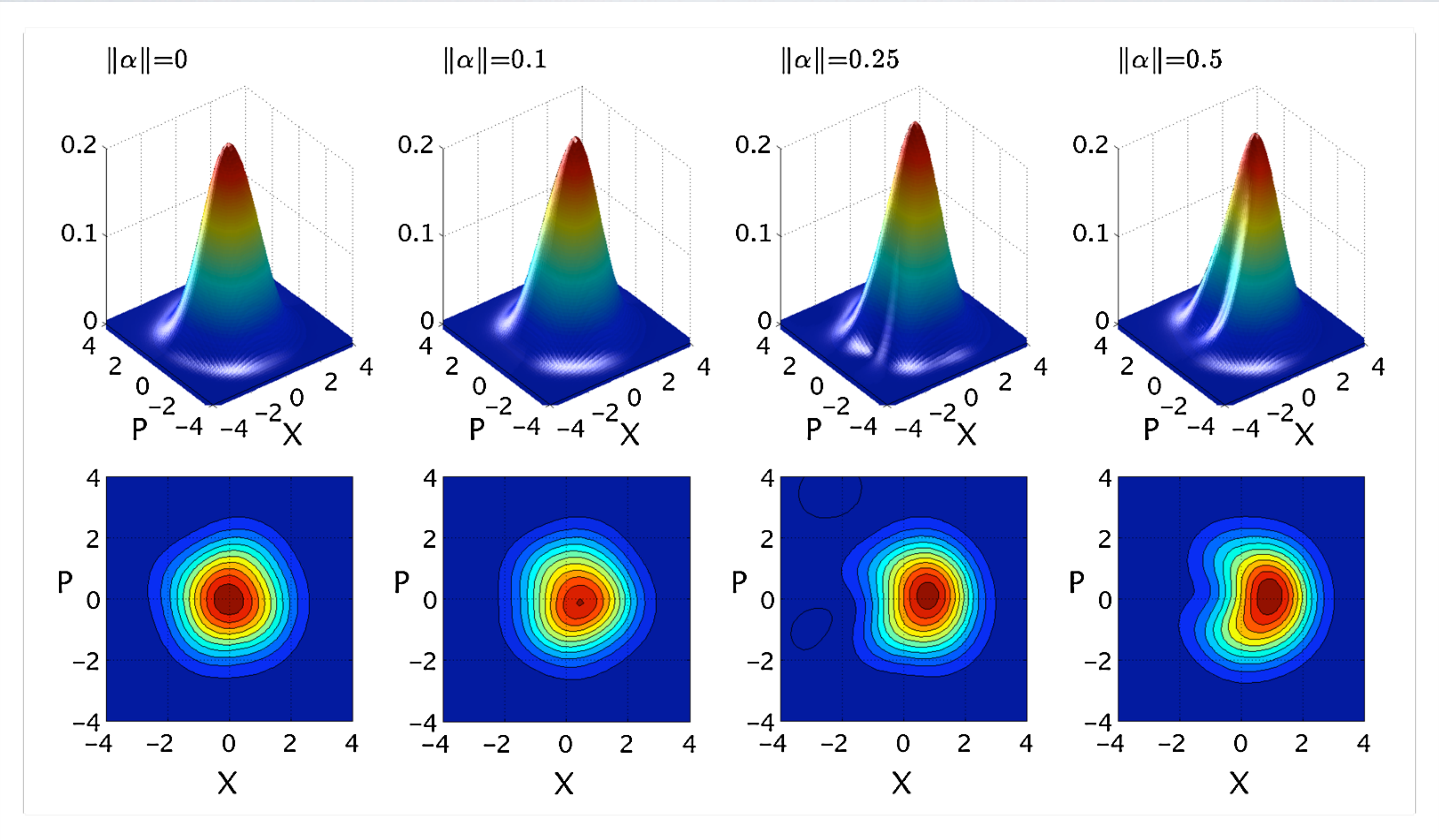
$\simeq |g\alpha\rangle$

$$g = \frac{\sqrt{1-r^2}}{r}$$

- Output state truncated at 1 photon
- Good approximation for small amplitude

THE NOISELESS AMPLIFIER

Experimental implementation (quantum scissors)

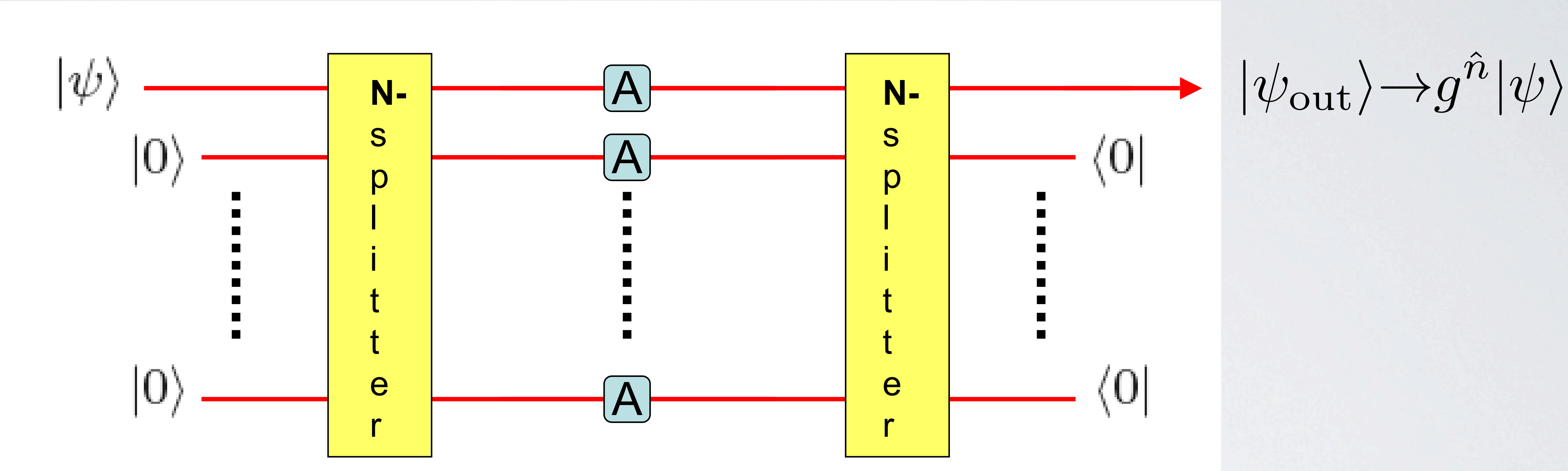


- Good proof of principle

PRL **104**, 123603 (2010)

HOW TO OBTAIN (ALMOST) $g^{\hat{n}}$?

Quantum scissors



arXiv:0809.0326 (2008)

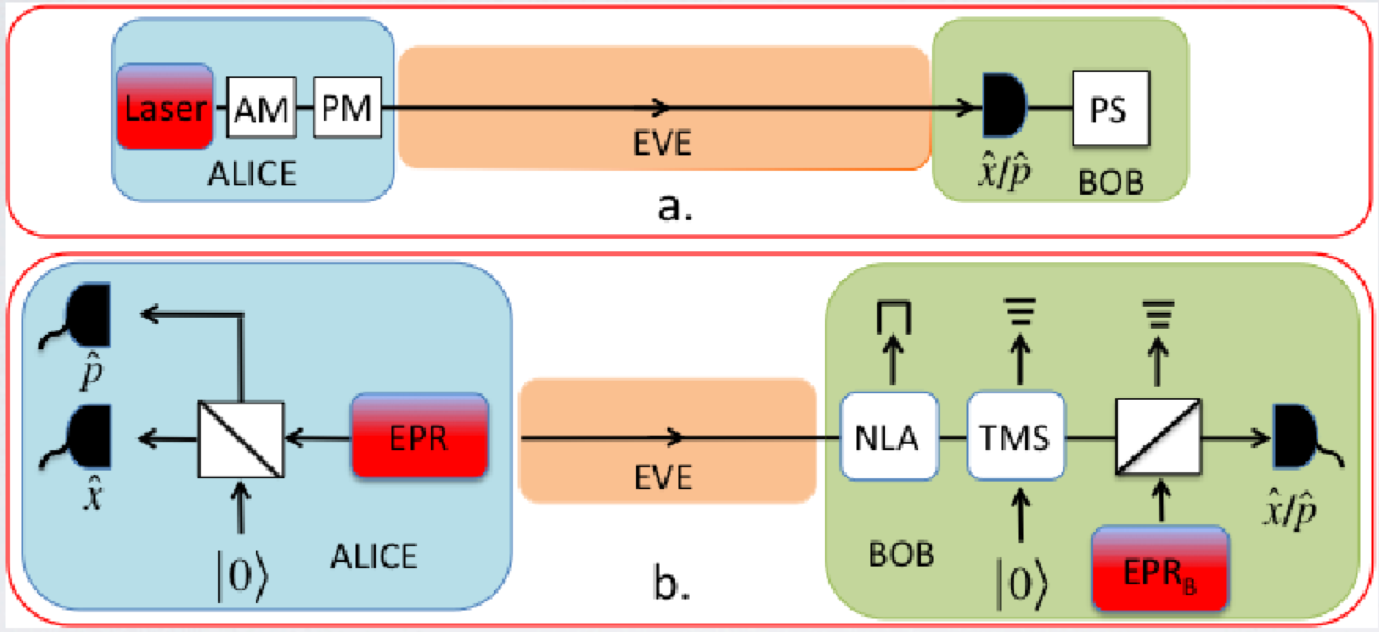
Polynomial approximation

$$g^{\hat{n}} = \lim_{N\to\infty} \sum_{k=0}^{N} \frac{(\ln g)^k}{k!} \hat{\boldsymbol{n}}^k$$

PRA 80, 053822 (2009)

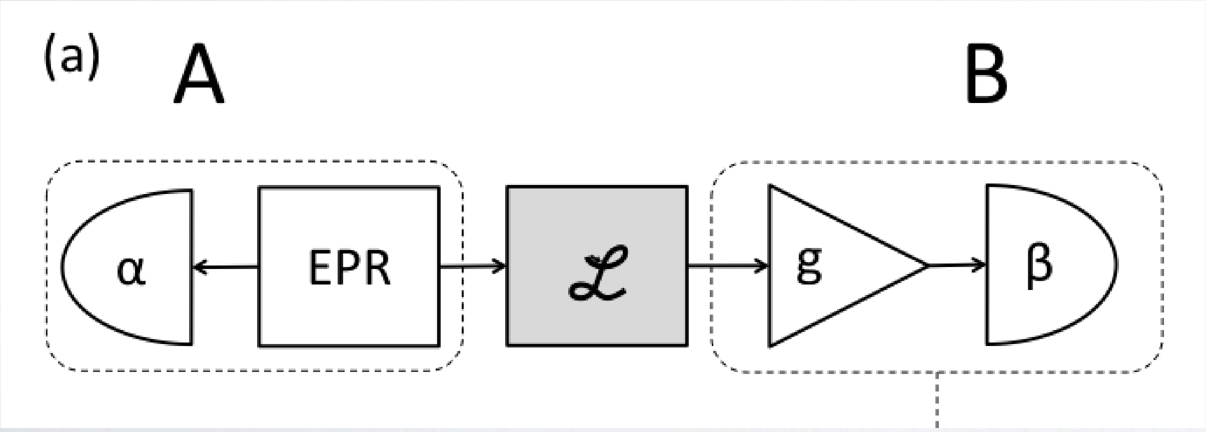
VIRTUAL IMPLEMENTATION USING POST-SELECTION

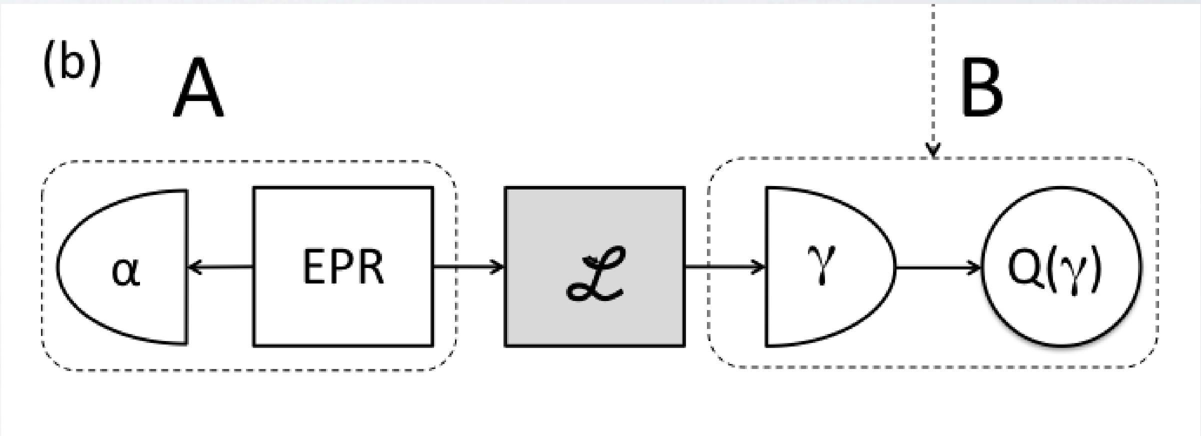
Homodyne detection for Bob



N. Walk et al., arXiv:1206.0936 (2012)

Heterodyne detection for Bob





J. Fiurasek and N. J. Cerf, arXiv:1205.6933 (2012)

THE NOISELESS AMPLIFIER

Theoretical implementation

Quantum scissors
T.C.Ralph and A.P.Lund, arXiv:0809.0326 (2008)

Weak values
arXiv:0903.4181 (2009)

Photon addition and subtraction
PRA 80, 053822 (2009)

Phase amplification
PRA 81, 022302 (2010)

Experimental implementation

Quantum scissors
PRL 104, 123603 (2010)
Nat. Photon. 4, 316 (2010)

Short review of the experiments
Laser Physics Letters 8, 411–417 (2011)

Photon addition and subtraction
Nature Photon. 5, 52 (2011)

Phase amplification
Nat. Phys. 6, 767 (2010)

Applications

This talk
PRA 86, 012327 (2012)

Virtual implementation and QKD
arXiv:1205.6933 (2012)
arXiv:1206.0936 (2012)

Optical loss suppression
arXiv:1206.2852 (2012)

Error correction
PRA 84, 022339 (2011)

Cloning of coherent states
PRA 86, 010305 (2012)

No violation of causality
PRA 86, 012324 (2012)

III. Improvement of CV-QKD performances with the NLA

MOTIVATIONS

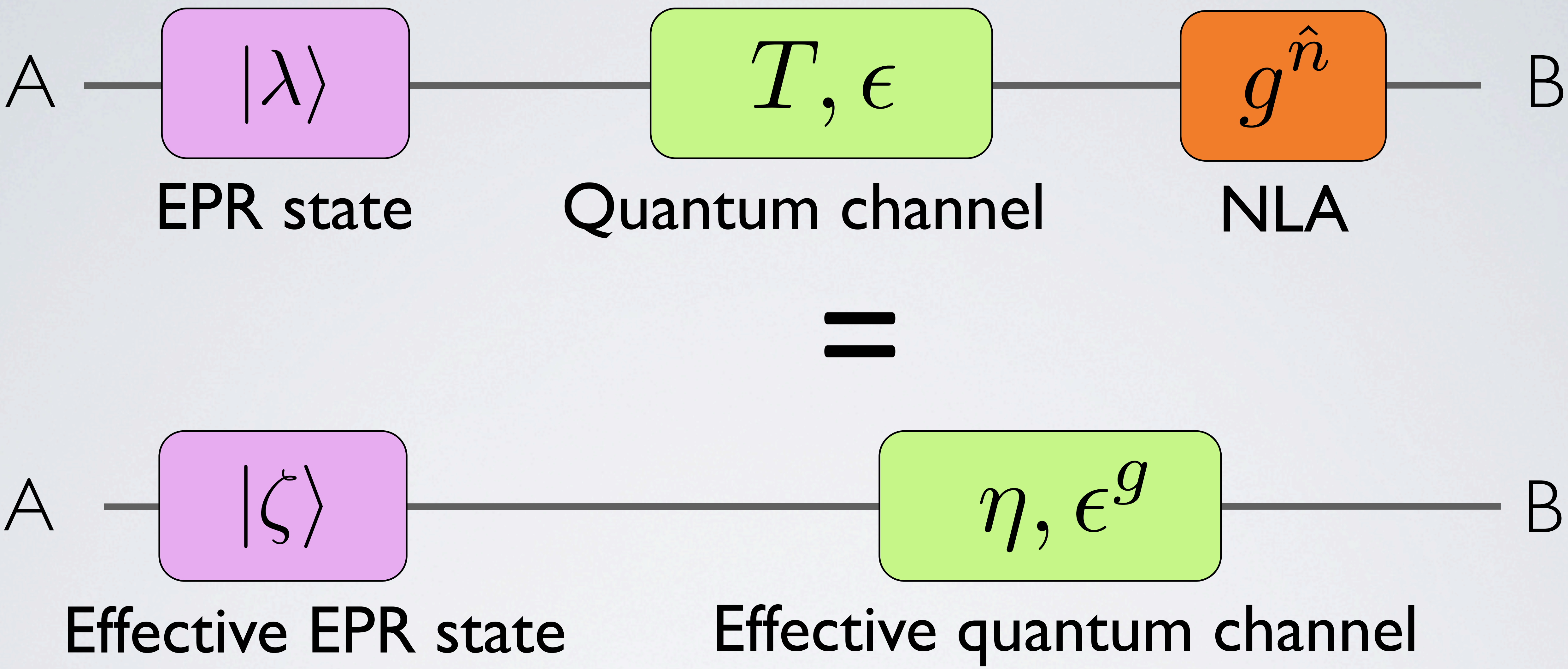
How does Bob's NLA improve

- the maximum distance of transmission?
- the maximum tolerable noise?
- the key rate?

Assumption:

- Linear lossy and noisy Gaussian channel
  - transmittance T
  - added noise ϵ

EQUIVALENT SYSTEM



Covariance matrices

$$\Gamma_{\mathrm{AB}}(\lambda, T, \epsilon, g) = \Gamma_{\mathrm{AB}}(\zeta, \eta, \epsilon^g, g{=}1)$$

EFFECTIVE PARAMETERS

Effective EPR parameter ζ

$$\zeta = \lambda \sqrt{\frac{(g^2-1)(\epsilon-2)T-2}{(g^2-1)\epsilon T-2}}$$

- The NLA increases the entanglement
- ζ depends linearly on λ

Physical constraint

$$0 \leq \zeta < 1 \Rightarrow 0 \leq \lambda < \left(\sqrt{\frac{(g^2-1)(\epsilon-2)T-2}{(g^2-1)\epsilon T-2}}\right)^{-1}$$

EFFECTIVE PARAMETERS

Effective transmittance η and noise ϵ^g

$$\eta = \frac{g^2 T}{\left(g^2-1\right) T\left[\frac{1}{4}\left(g^2-1\right)\left(\epsilon-2\right)\epsilon T-\epsilon+1\right]+1}$$

$$\epsilon^g = \epsilon + \frac{1}{2}\left(g^2-1\right)\left(2-\epsilon\right)\epsilon T$$

- The NLA increases the transmittance and the noise
- No dependence on λ

Physical constraints

$$\left.\begin{array}{l} 0 \leq \eta \leq 1 \\ 0 \leq \epsilon^g \end{array}\right\} \Rightarrow \left\{\begin{array}{l} \epsilon \leq 2 \\ g \leq g_{max}(T,\epsilon) \end{array}\right.$$

SECRET KEY RATE

Key rate without the NLA

$$\Delta I(\lambda, T, \epsilon, \beta) = \beta I_{AB}(\lambda, T, \epsilon) - \chi_{BE}(\lambda, T, \epsilon)$$

Key rate with the NLA

= Key rate with the effective parameters without the NLA, weighted by the probability of success P_{suc}

$$\Delta I_{\mathrm{NLA}}(\lambda, T, \epsilon, \beta) = P_{\mathrm{suc}} \Delta I(\zeta, \eta, \epsilon^g, \beta)$$

Alice optimizes the variance modulation to maximize the key rate

PROBABILITY OF SUCCESS

Optimistic upper bound

$$P_{\mathrm{suc}} \leq \frac{1}{g^2}$$

Remarks on the probability of success

- Depends on the physical implementation
- Realistic probability of success may be much smaller
- Acts simply as a scaling factor
- Doesn't change the positivity or negativity of a key rate

IMPROVEMENT OF PERFORMANCES

Strong losses regime $(T << 1, \epsilon \neq 0)$

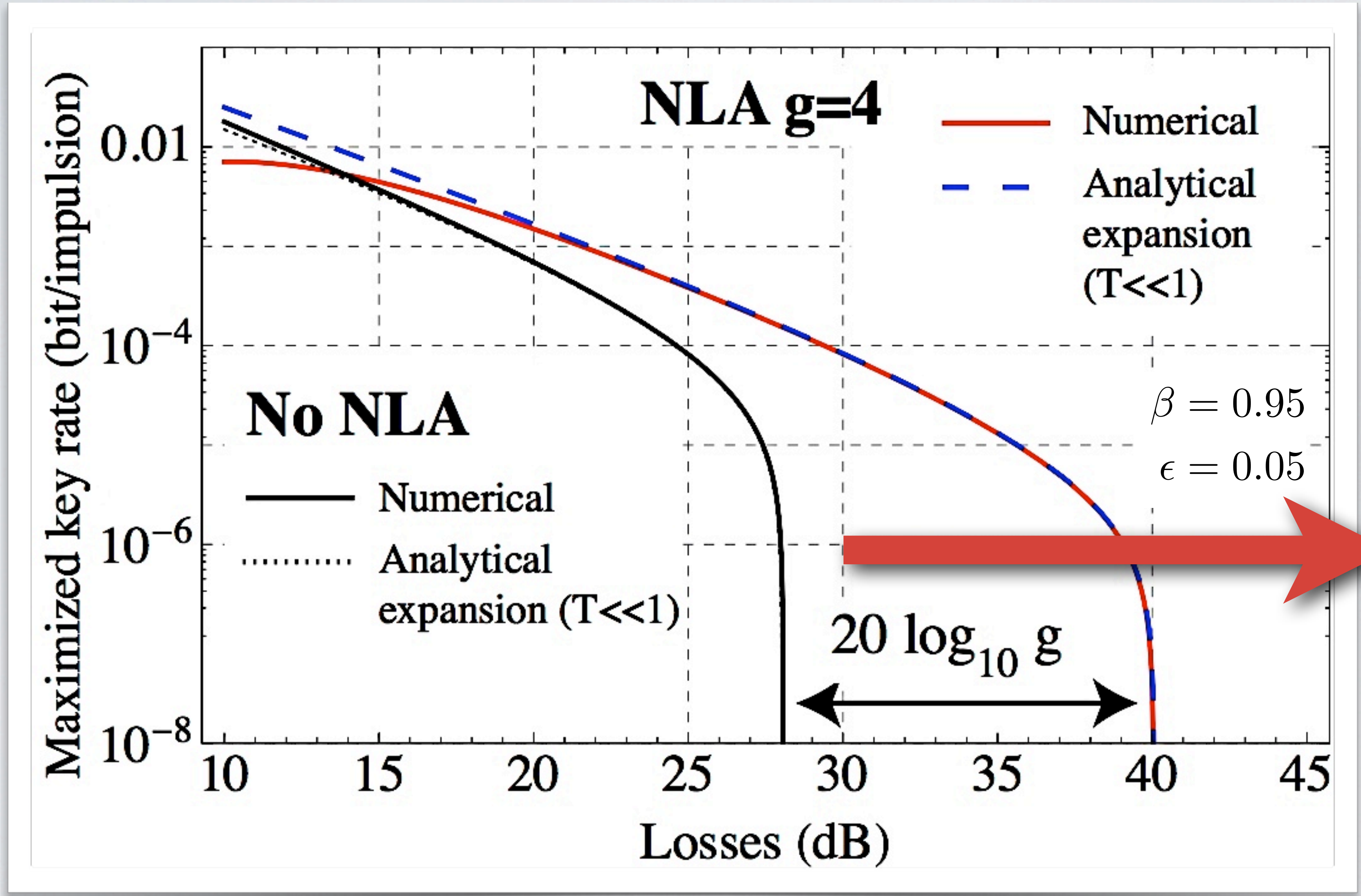
- Without the NLA: minimum value of transmittance T_{lim}
- With the NLA:

$$T_{\text{lim}}^{\text{NLA}} = \frac{1}{g^2} T_{\text{lim}}$$

Tolerable losses are increased by $20 \log_{10} g$ dB

The maximum distance of transmission can be arbitrarily increased by increasing the gain

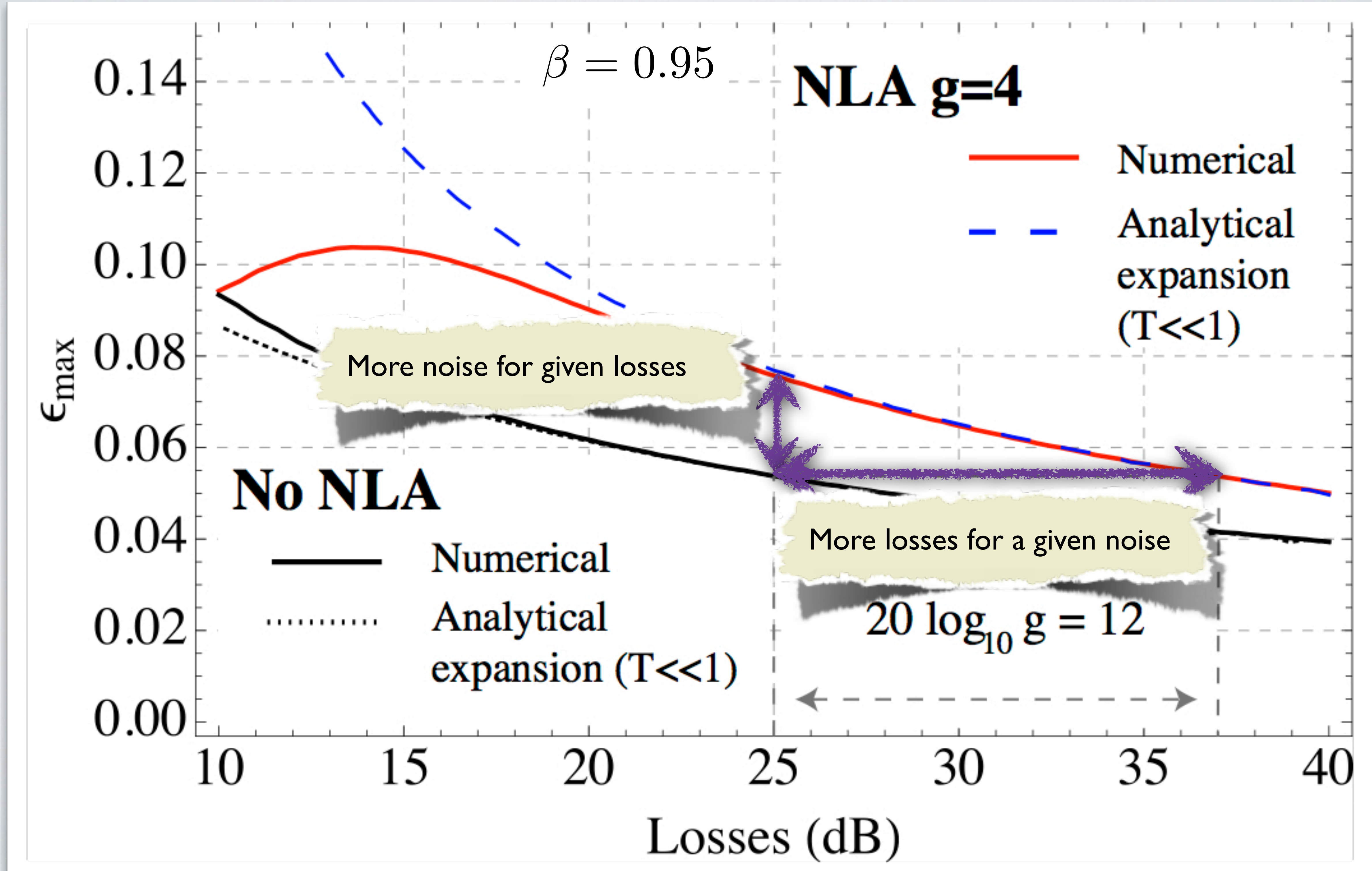
Maximized key rate



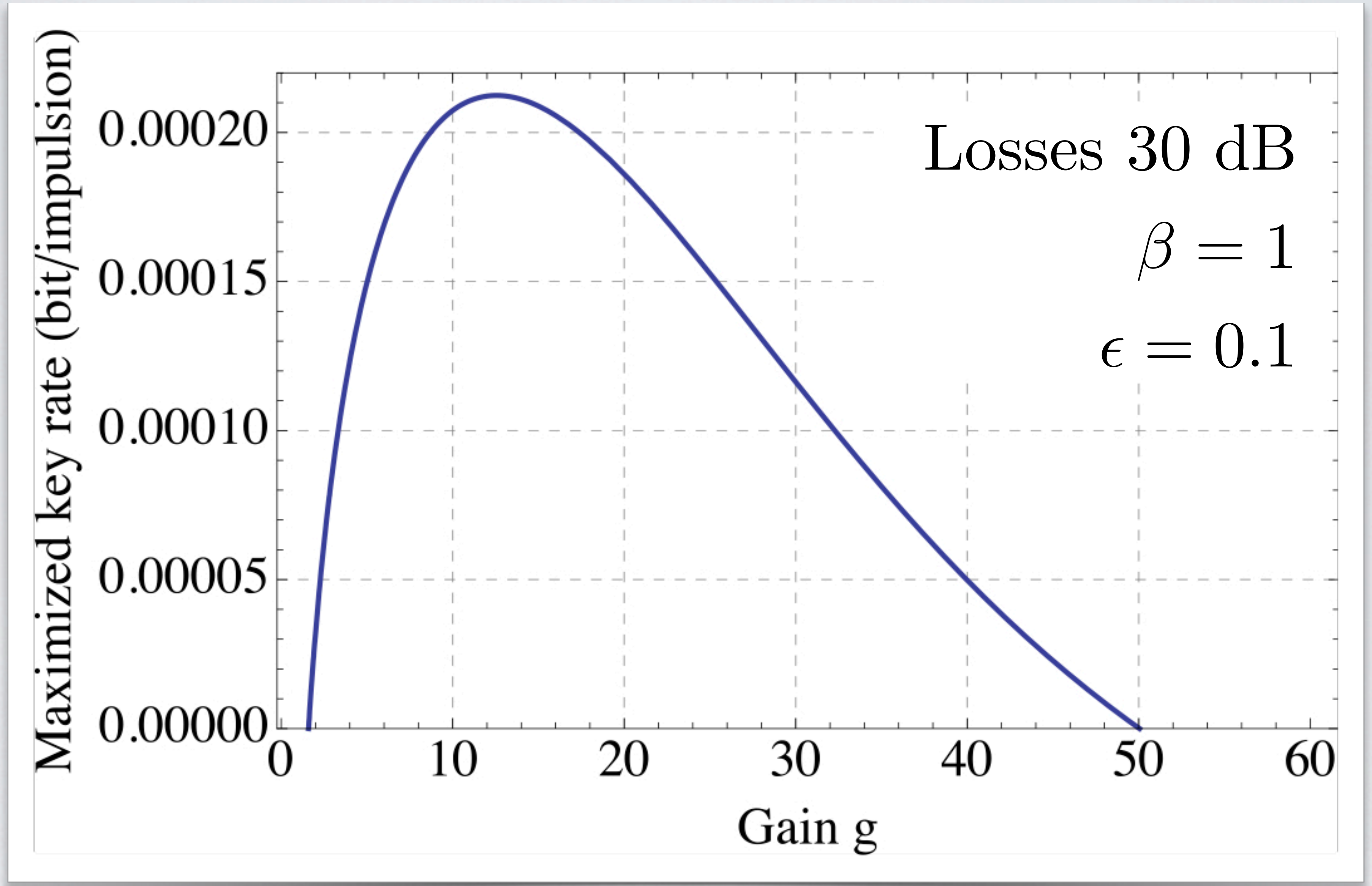
Increase of the maximum distance of transmission by increasing the gain

IMPROVEMENT OF PERFORMANCES

Maximum tolerable noise



Can we arbitrarily increase the key rate? No...



- Optimal value of the gain
- If the gain if too important, the effective noise becomes too high

CONCLUSION

NLA in CV-QKD with a Gaussian lossy noisy channel

- Equivalent to an effective system without the NLA
- The maximum distance of transmission can be arbitrarily increased
- Improvement of the maximum tolerable noise
- Explicit formulas for GG02, same resultats for other CV-QKD protocols (same effective parameters)

Reference: R. Blandino *et al.*, PRA **86**, 012327 (2012)

THANK YOU

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OUR TEAM

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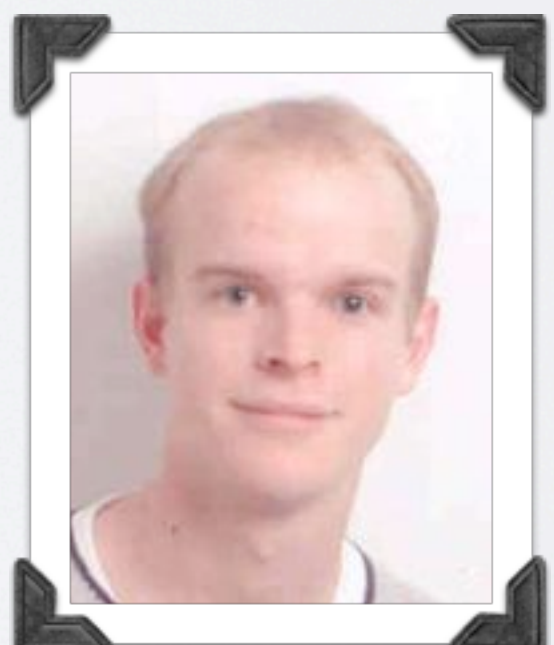

A. Leverrier