Authentication

Ueli Maurer

ETH Zurich

QCRYPT 2012, Singapore

Authentication

and more ...

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Three goals of this talk

1. Role of authentication in QKD

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2. Information-theoretically secure authentication

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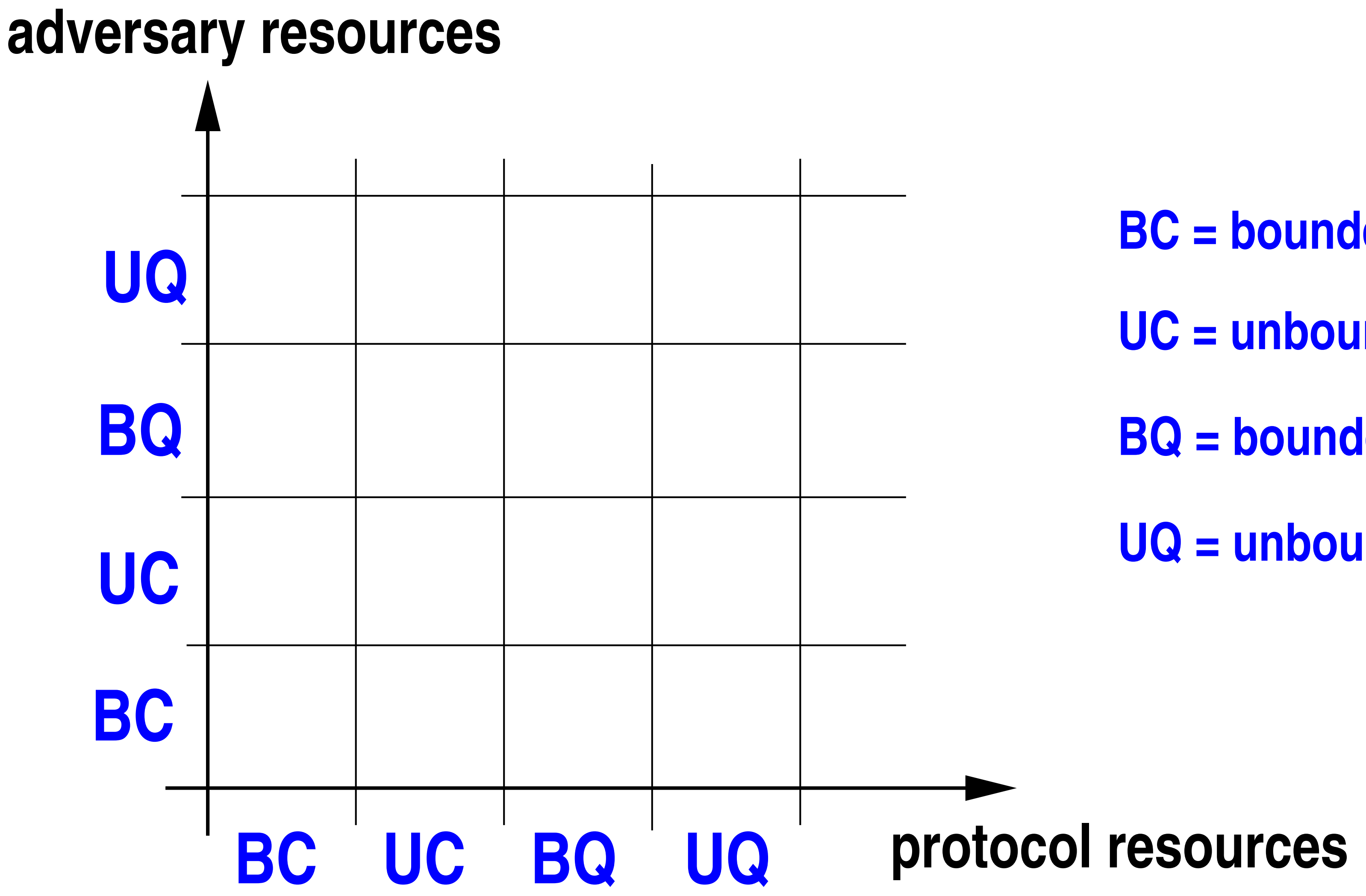
3. Constructive approach to cryptography

Three goals of this talk

1. Role of authentication in QKD
2. Information-theoretically secure authentication
3. Constructive approach to cryptography
   (joint work with Renato Renner)

Security types – a classification



BC = bounded classical

UC = unbounded classical

BQ = bounded quantum

UQ = unbounded quantum

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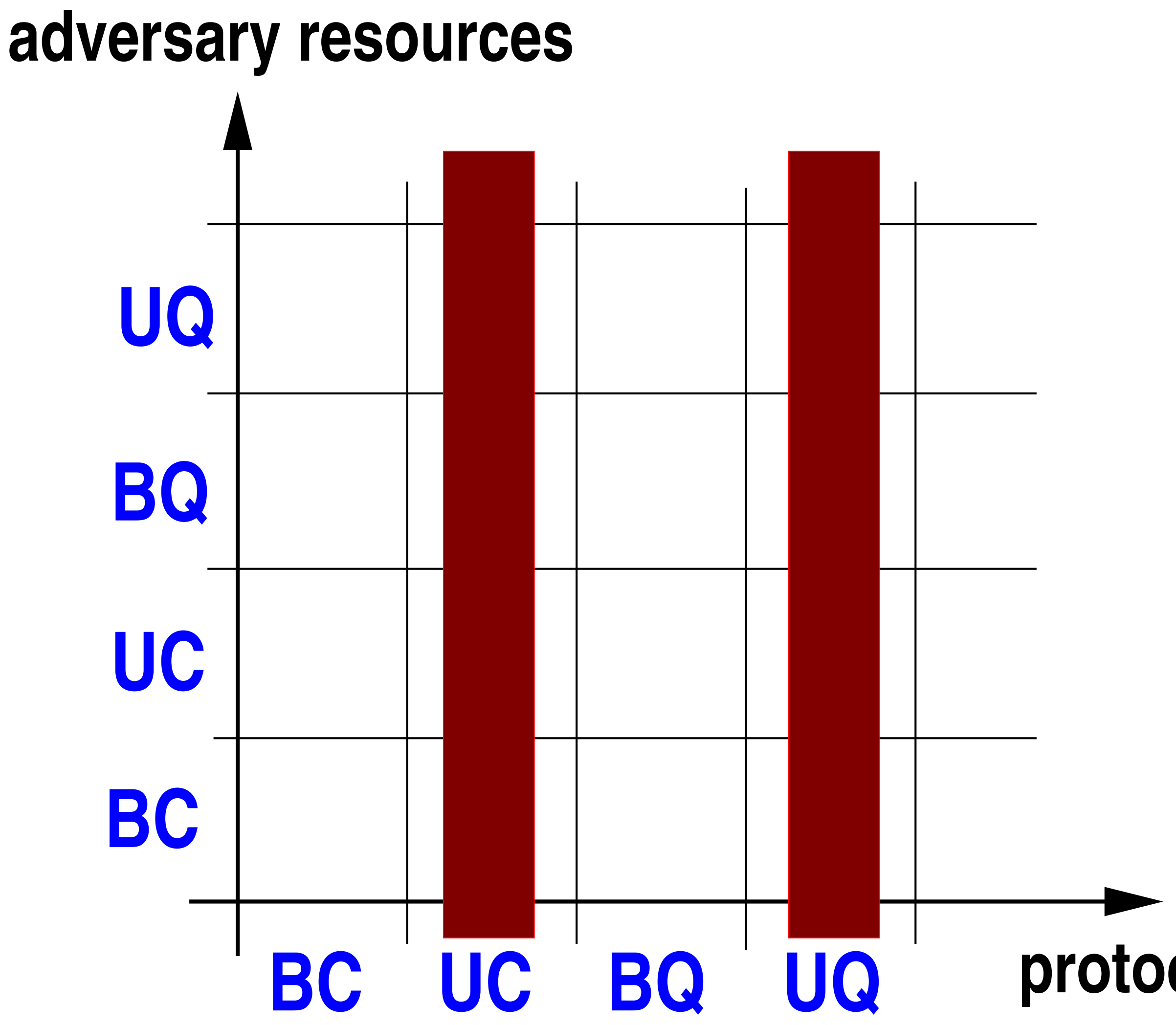

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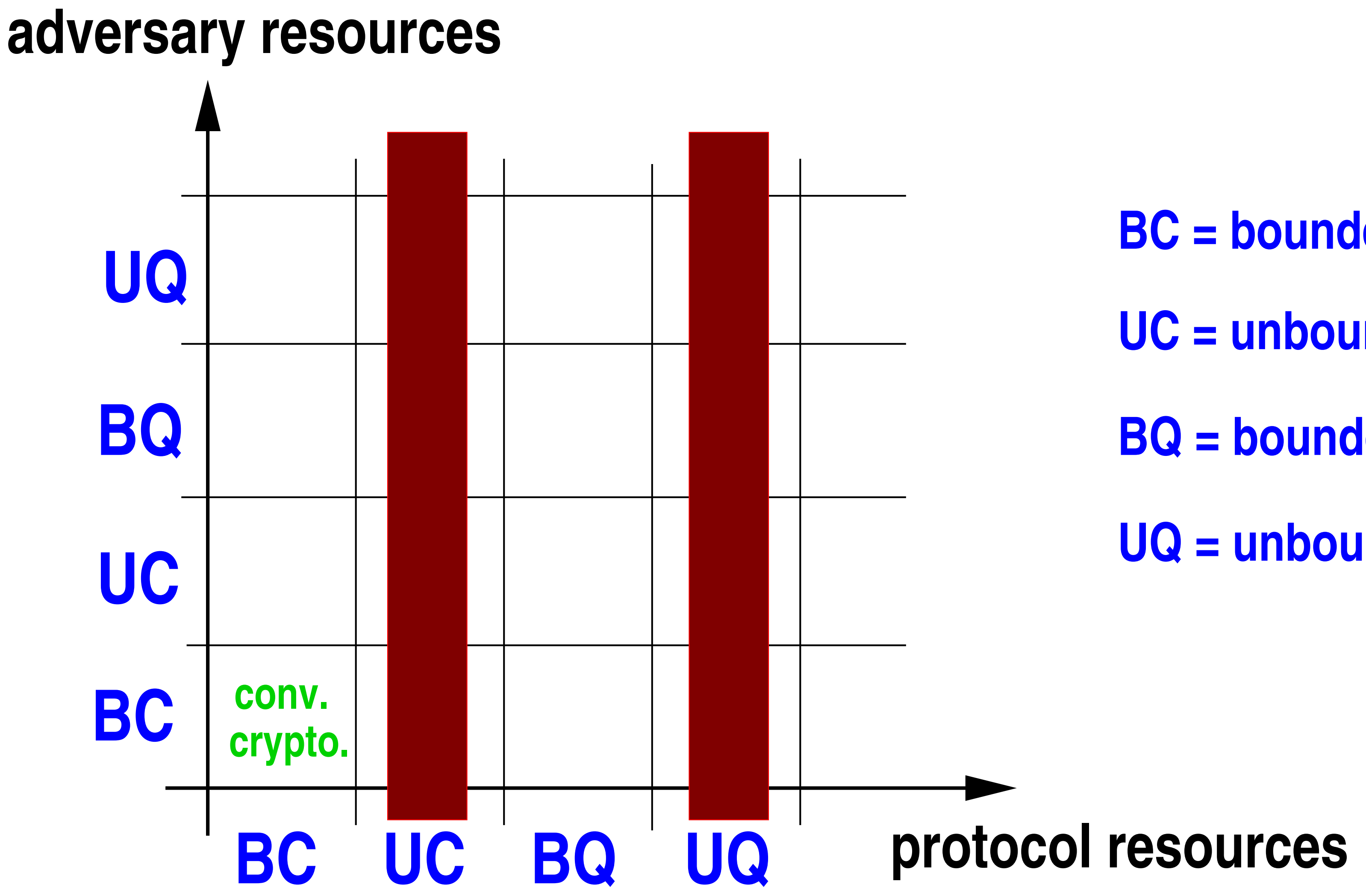

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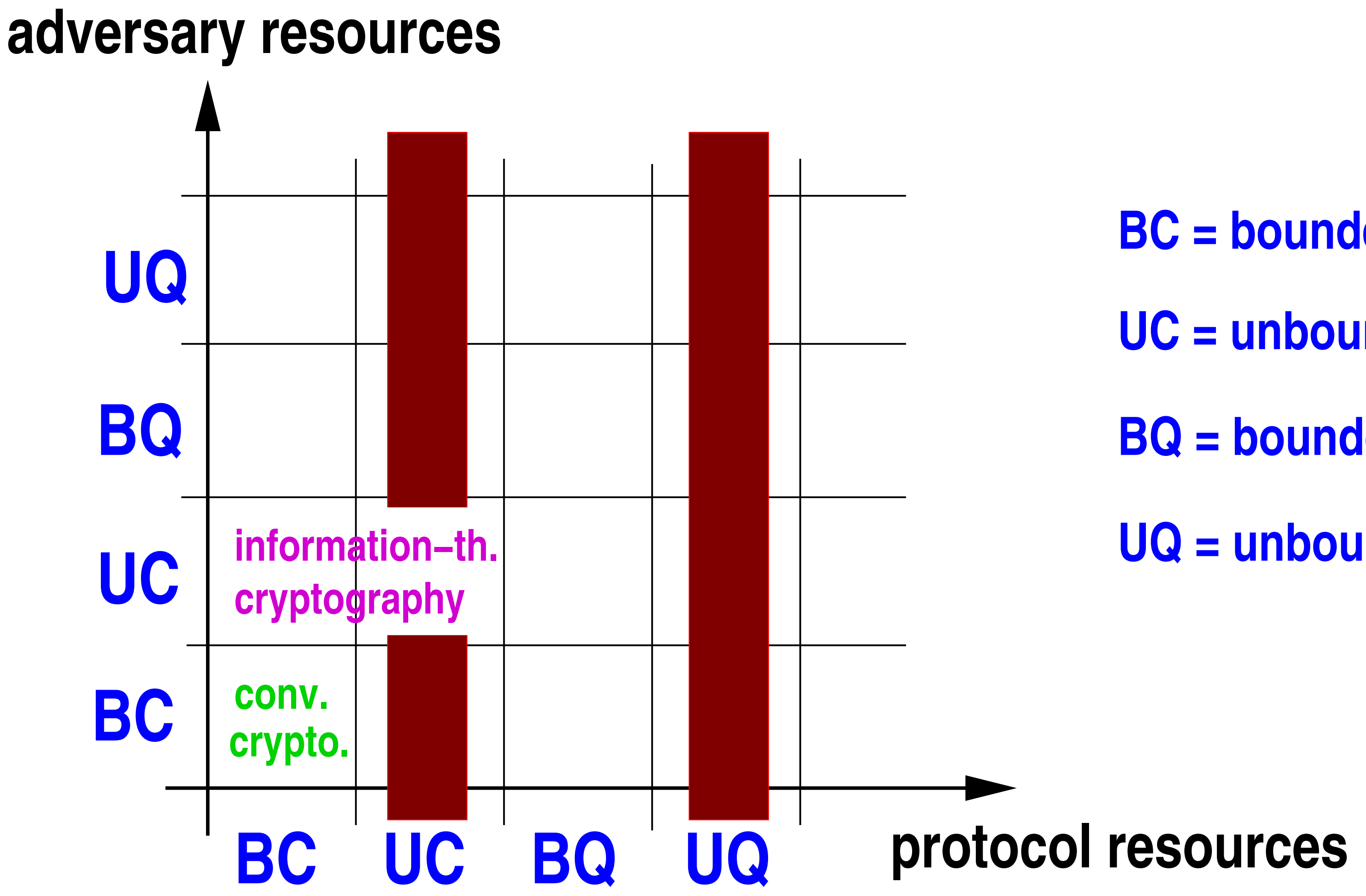

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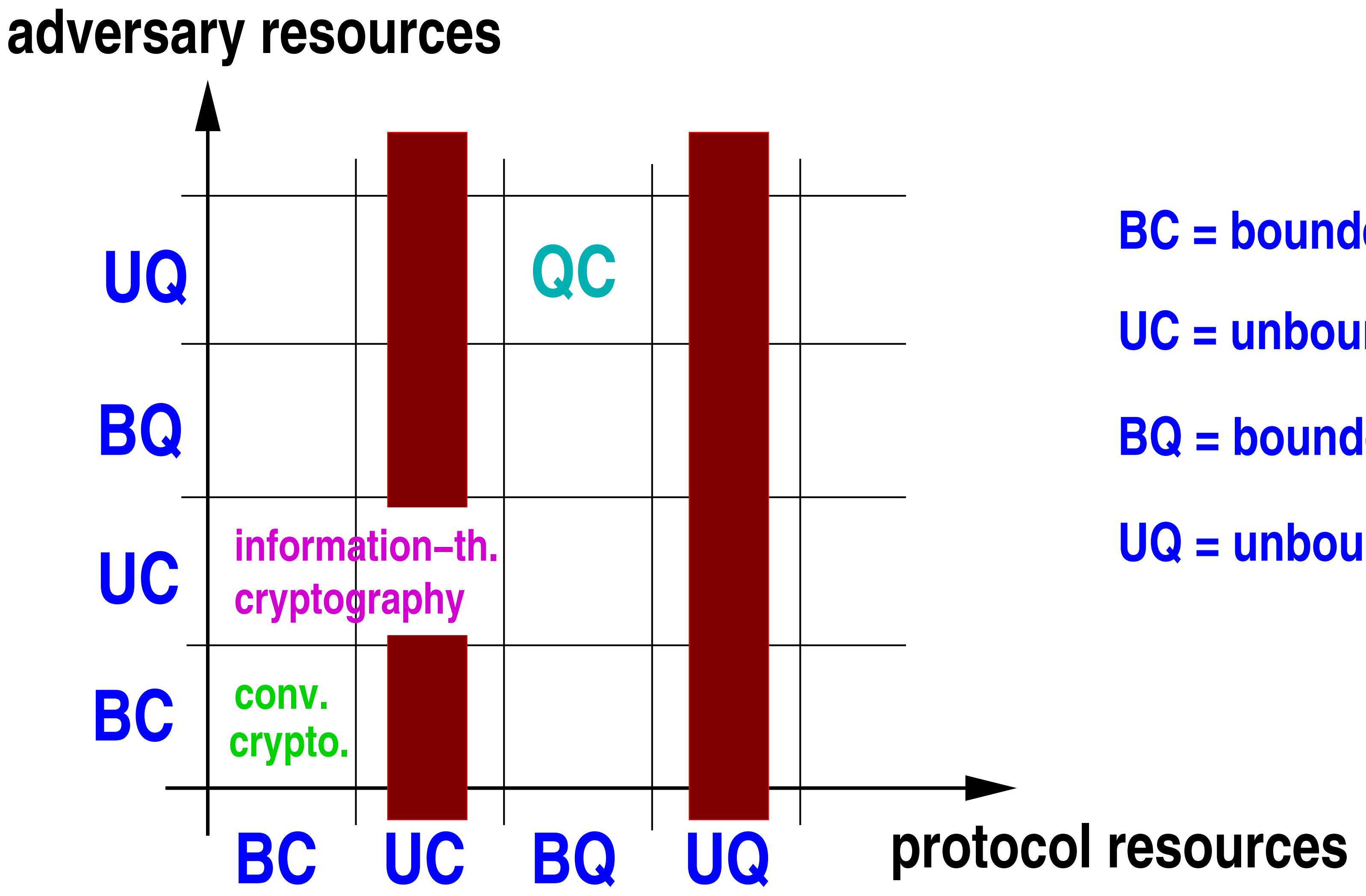

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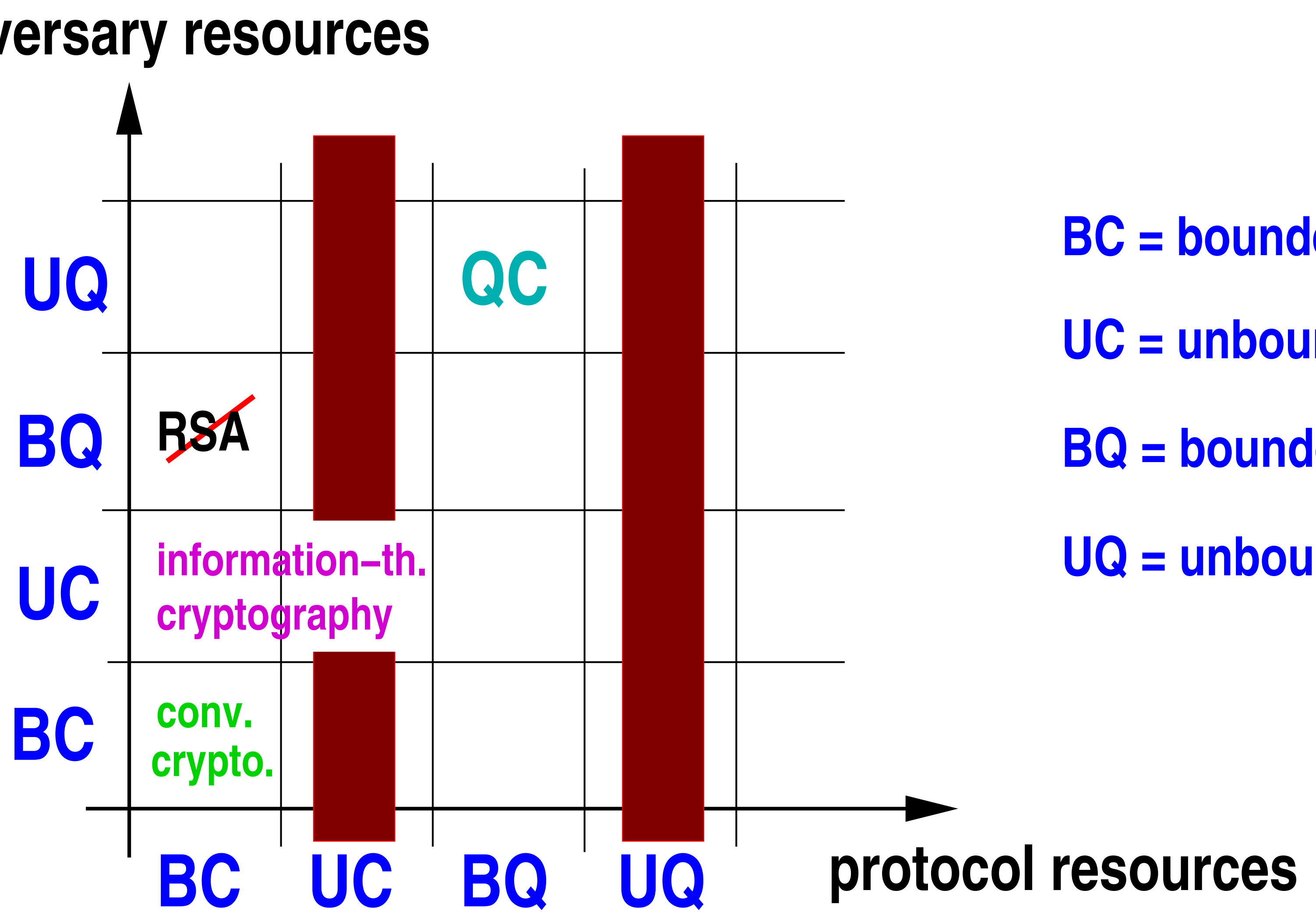

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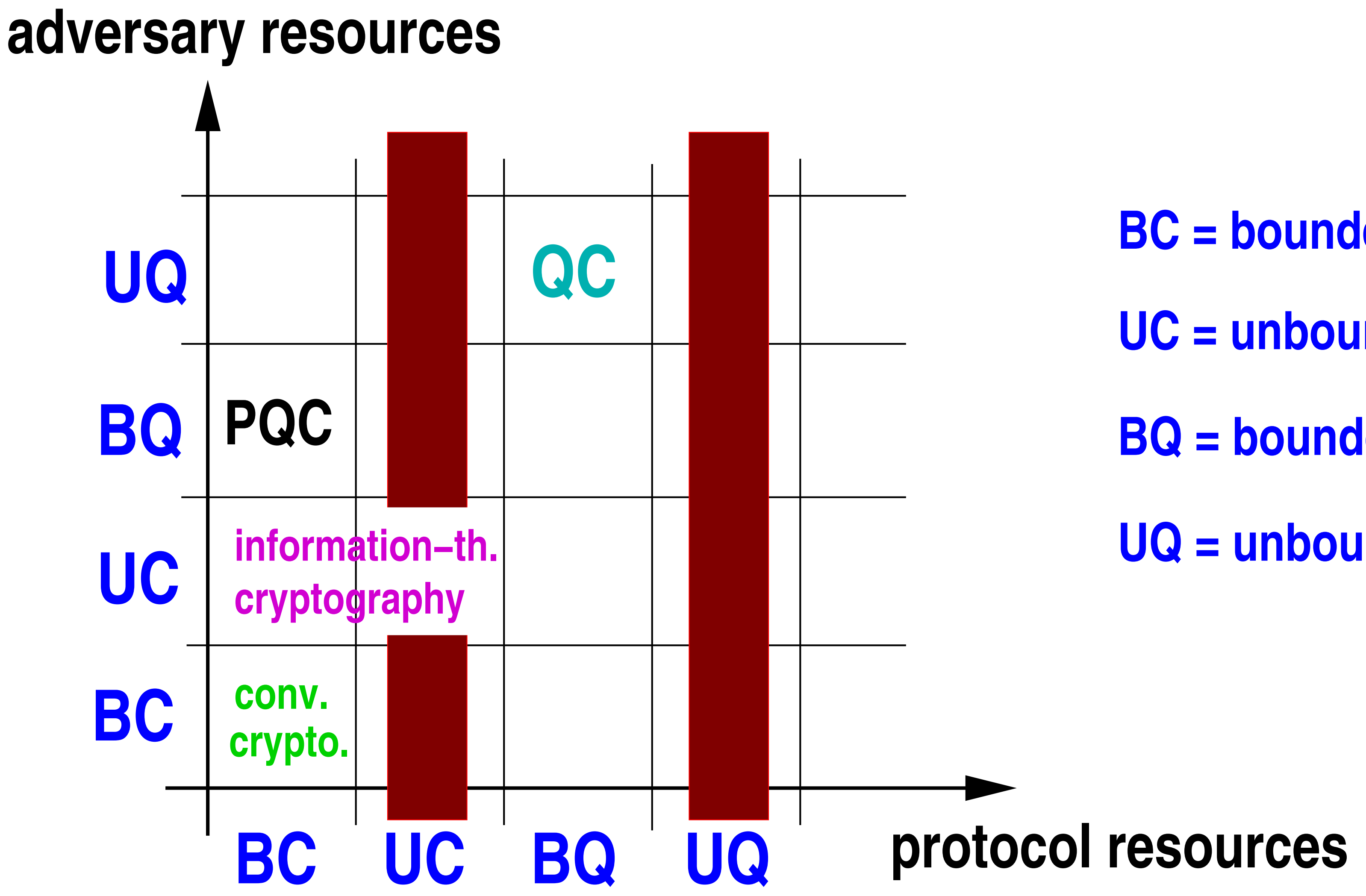

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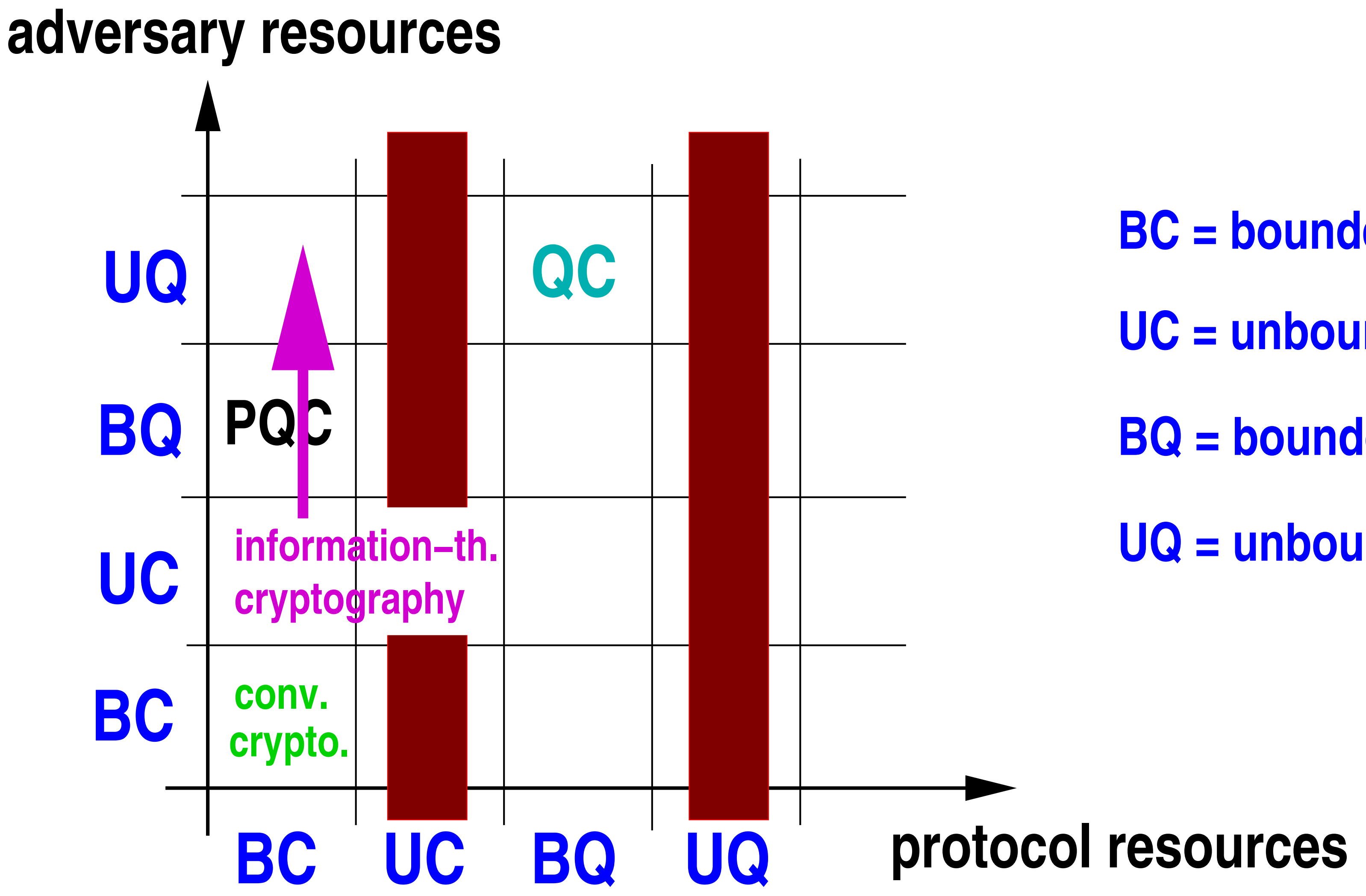

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Secrecy and authenticity

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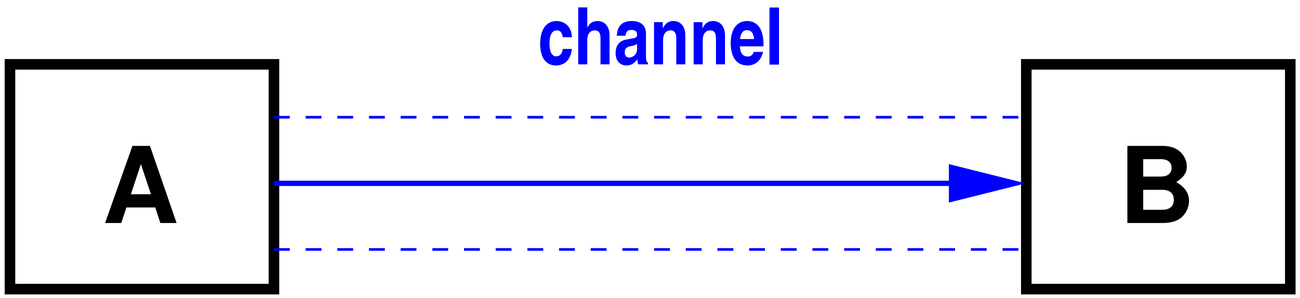

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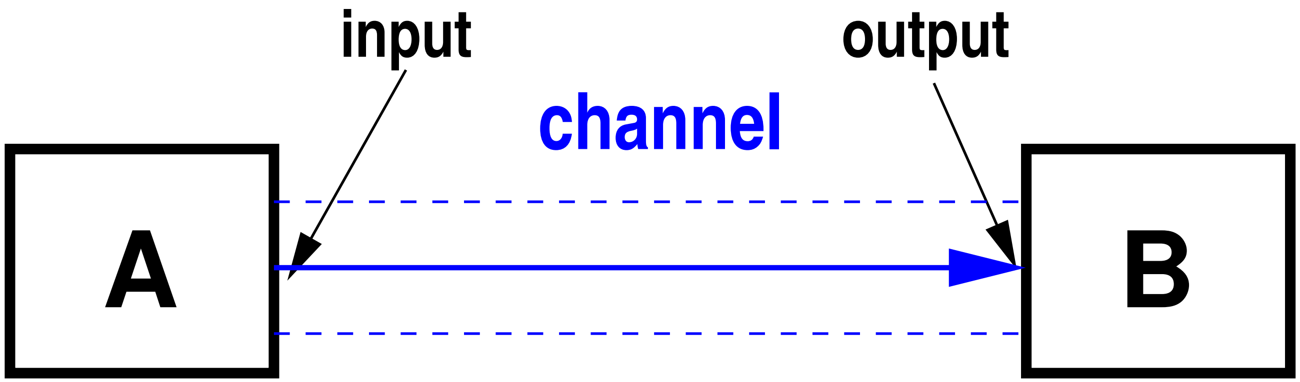

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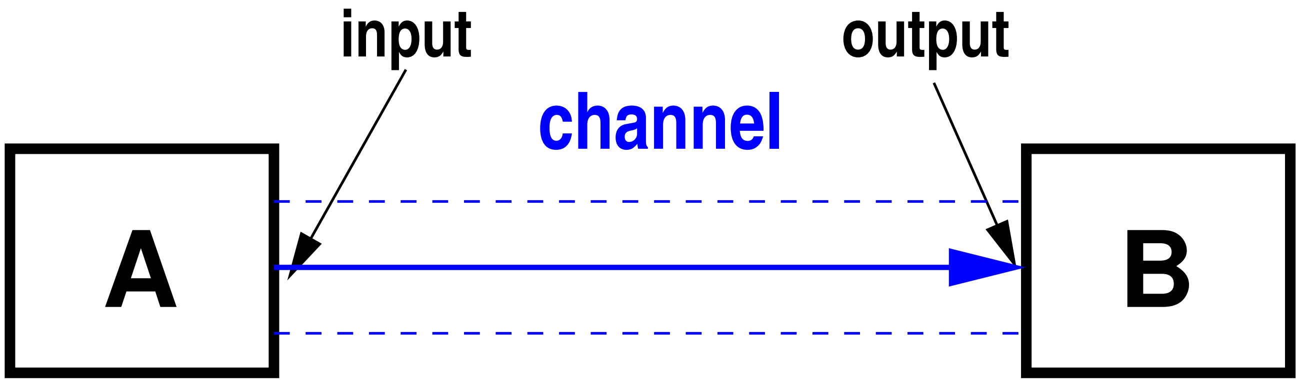
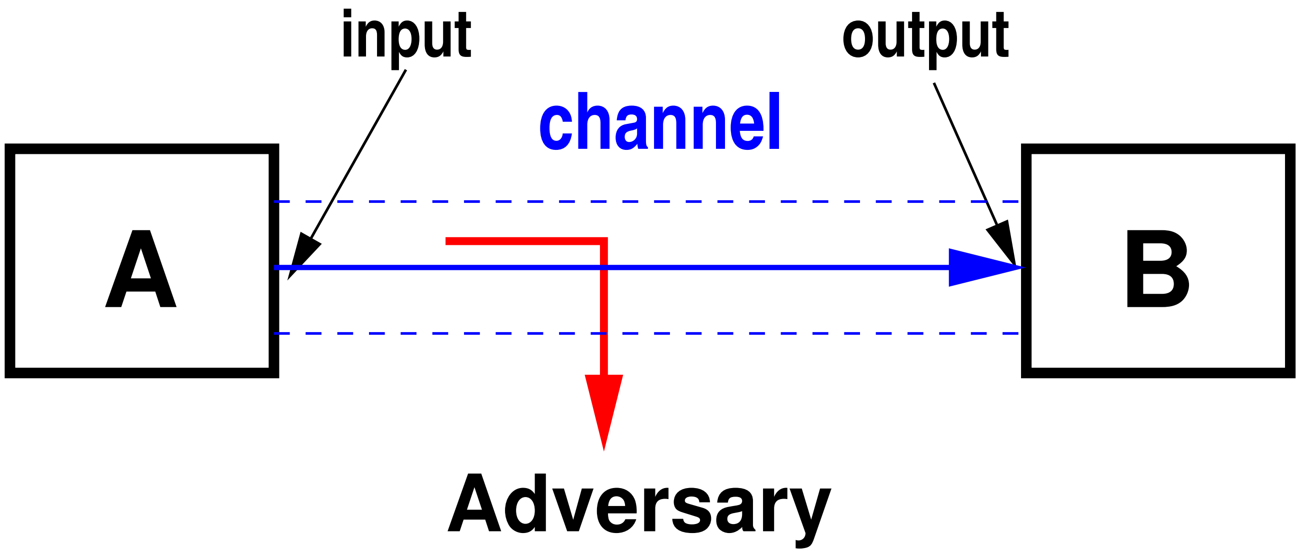


Adversary

Secrecy and authenticity



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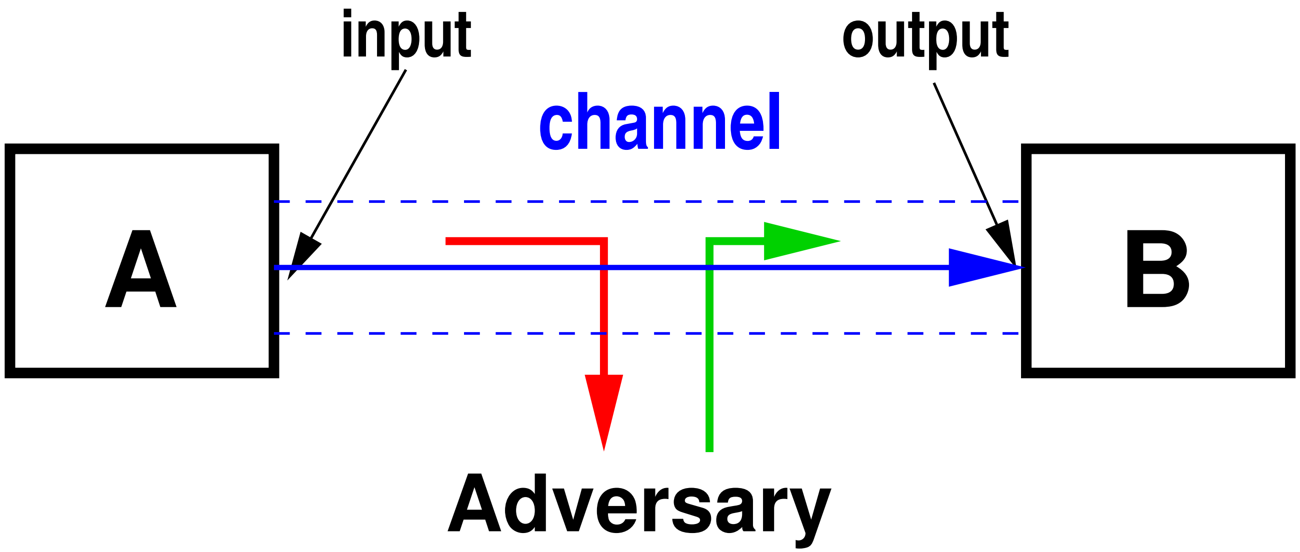

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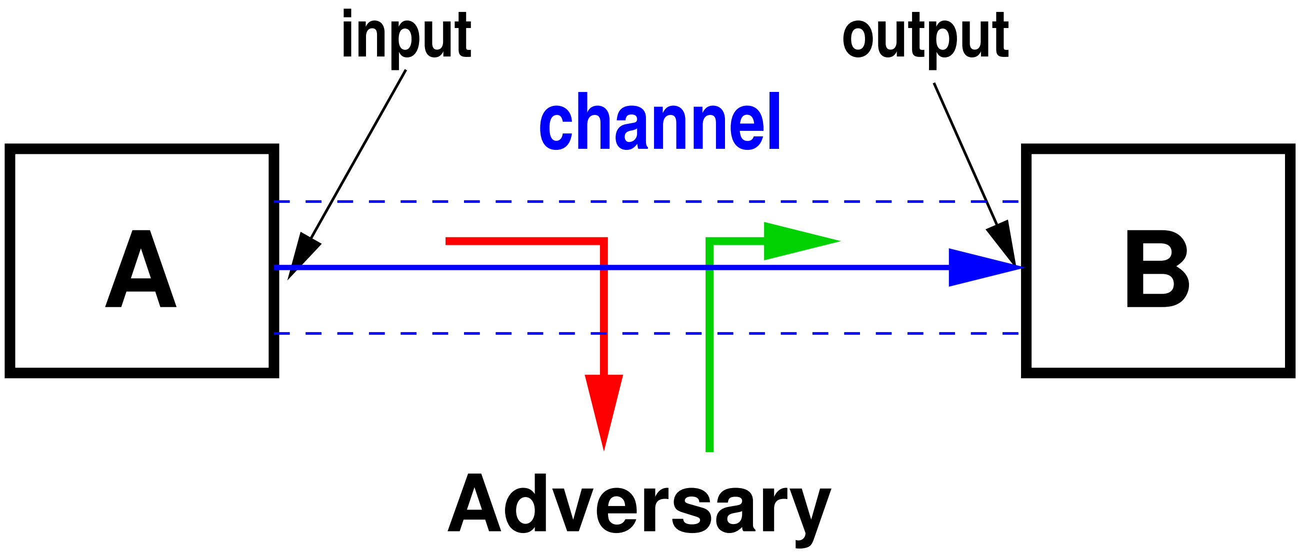
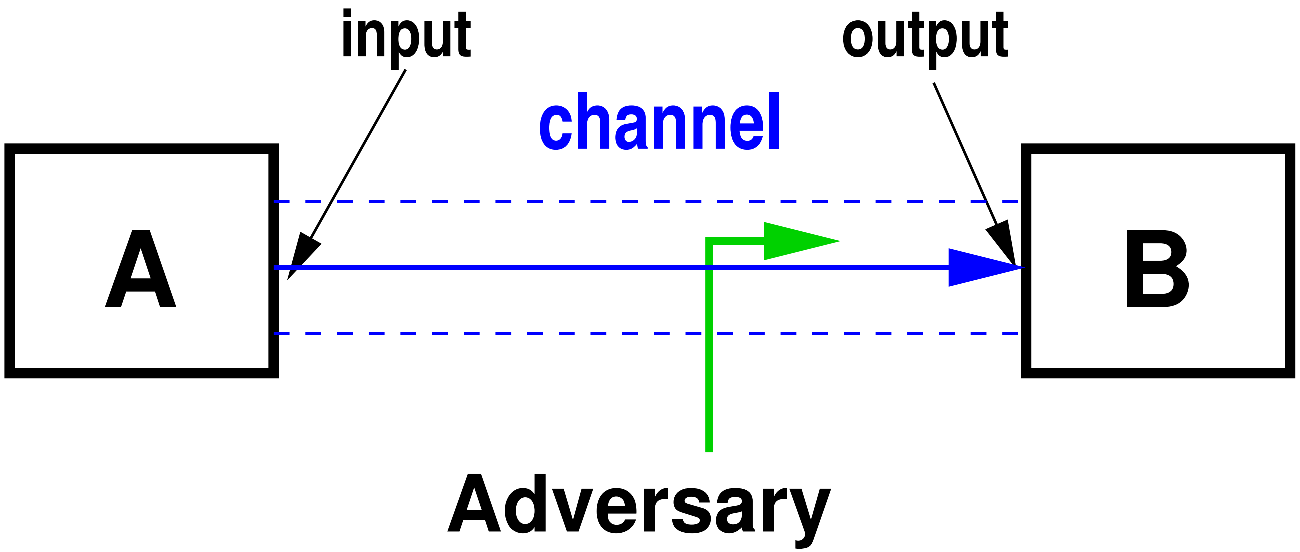


Two basic **independent / dual** security properties:

- **secrecy**
- **authenticity**

Secrecy and authenticity



Two basic **independent / dual** security properties:

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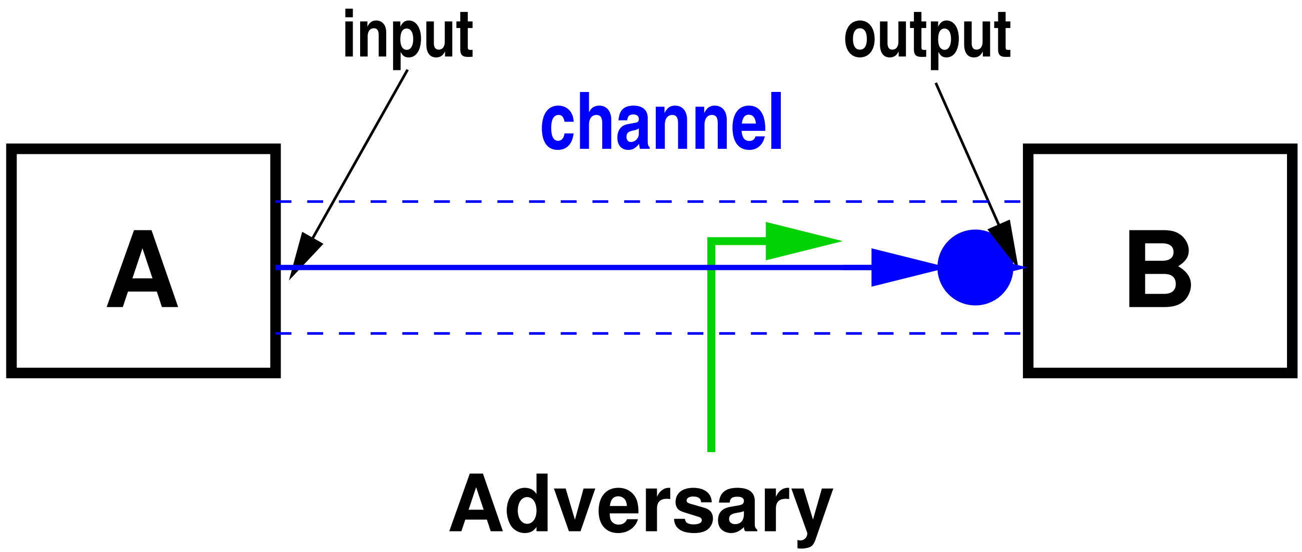
Secrecy and authenticity



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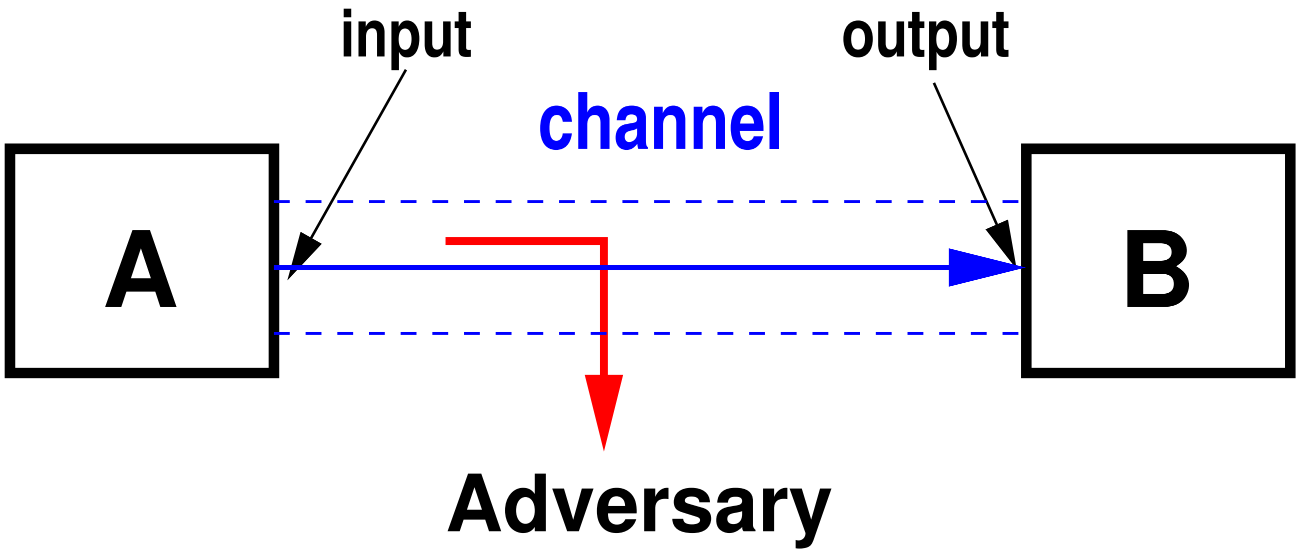
Secrecy and authenticity



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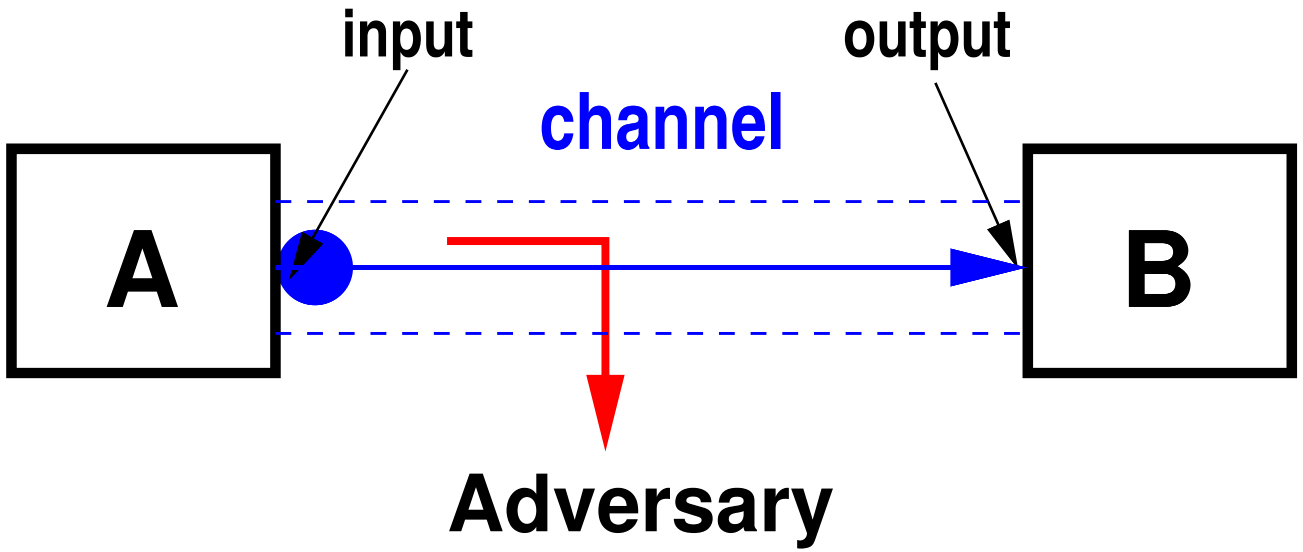

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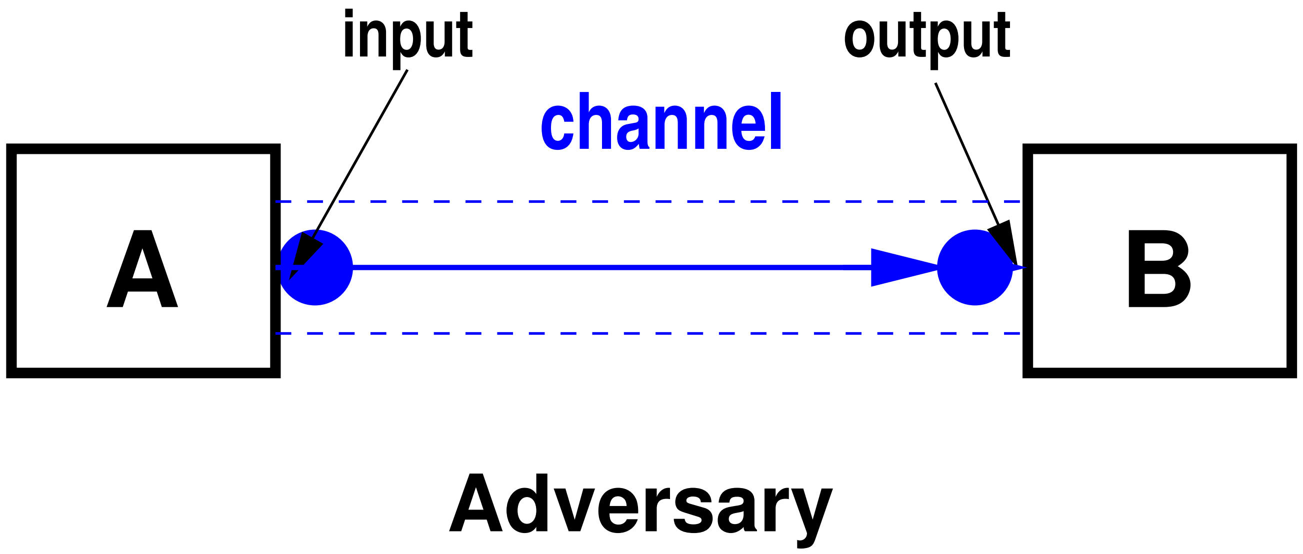

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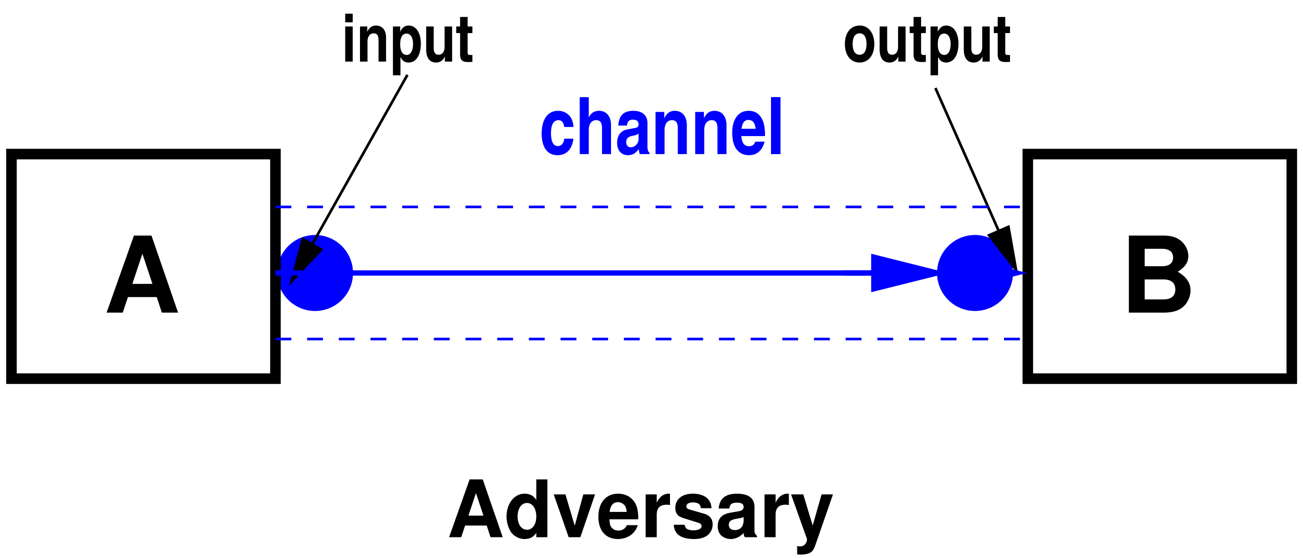
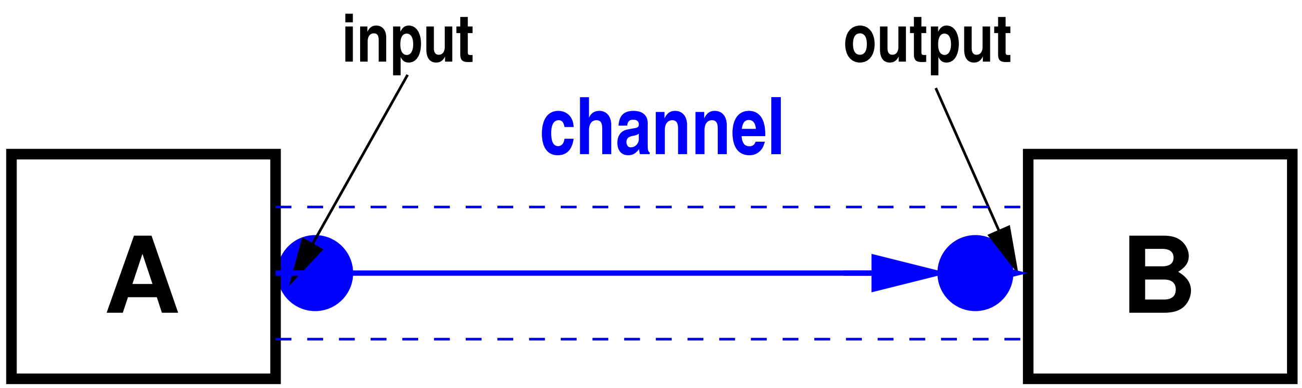
Secrecy and authenticity



$A \longrightarrow B$	(insecure) channel from A to B
$A \longrightarrow\!\bullet\ B$	secret channel from A to B
$A \bullet\!\!\longrightarrow B$	authentic channel from A to B
$A \bullet\!\!\longrightarrow\!\bullet\ B$	secure channel from A to B (secret and authentic)

Secrecy and authenticity



Adversary

$A \longrightarrow B$	(insecure) channel from A to B
$A \longrightarrow\!\!\bullet\ B$	secret channel from A to B
$A \bullet\!\!\longrightarrow B$	authentic channel from A to B
$A \bullet\!\!\longrightarrow\!\!\bullet\ B$	secure channel from A to B (secret and authentic)
$A \quad \bullet\!\!=\!\!=\!\!=\!\!\bullet \quad B$	secret key shared by A and B
$A \quad =\!\!=\!\!=\!\!\bullet \quad B$	one-sided key: A knows that at most B knows the key, but B does not know who holds the key.

The •-calculus (for channels and keys)

Calculus

- for the design and analysis of cryptographic protocols
- cryptographic scheme = security transformation
- precise semantics (later)
- security proof by composition

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Calculus

- for the design and analysis of cryptographic protocols
- cryptographic scheme = security transformation
- precise semantics (later)
- security proof by composition

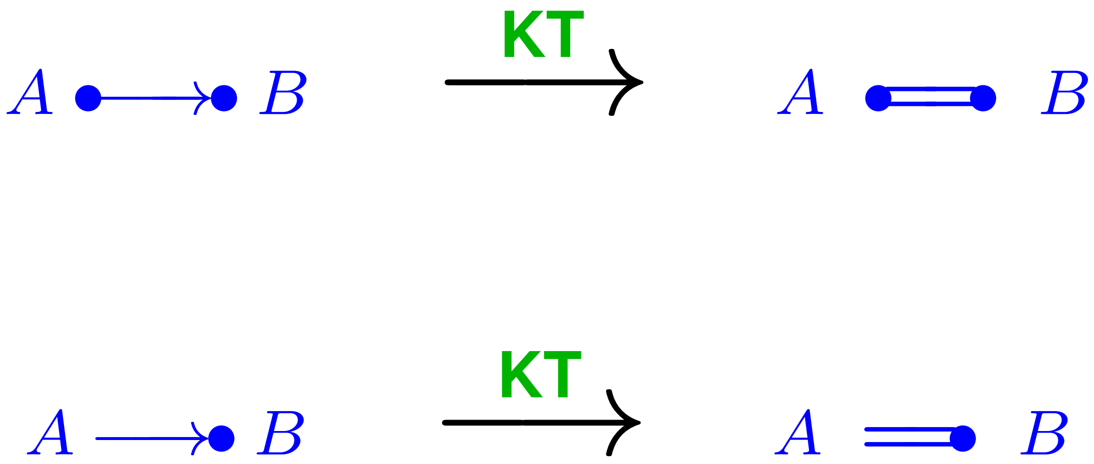
Illustrates:

- the relevant properties of various cryptographic systems
- limitations of cryptography
- role of protocols such as public-key certification
- role of trust
- necessary and sufficient conditions for key management in distributed systems

Key transport in •-calculus

$$A \bullet\!\!\longrightarrow\!\!\bullet\, B \qquad \xrightarrow{\textbf{KT}} \qquad A \bullet\!\!=\!\!=\!\!\bullet\, B$$

Key transport in •-calculus



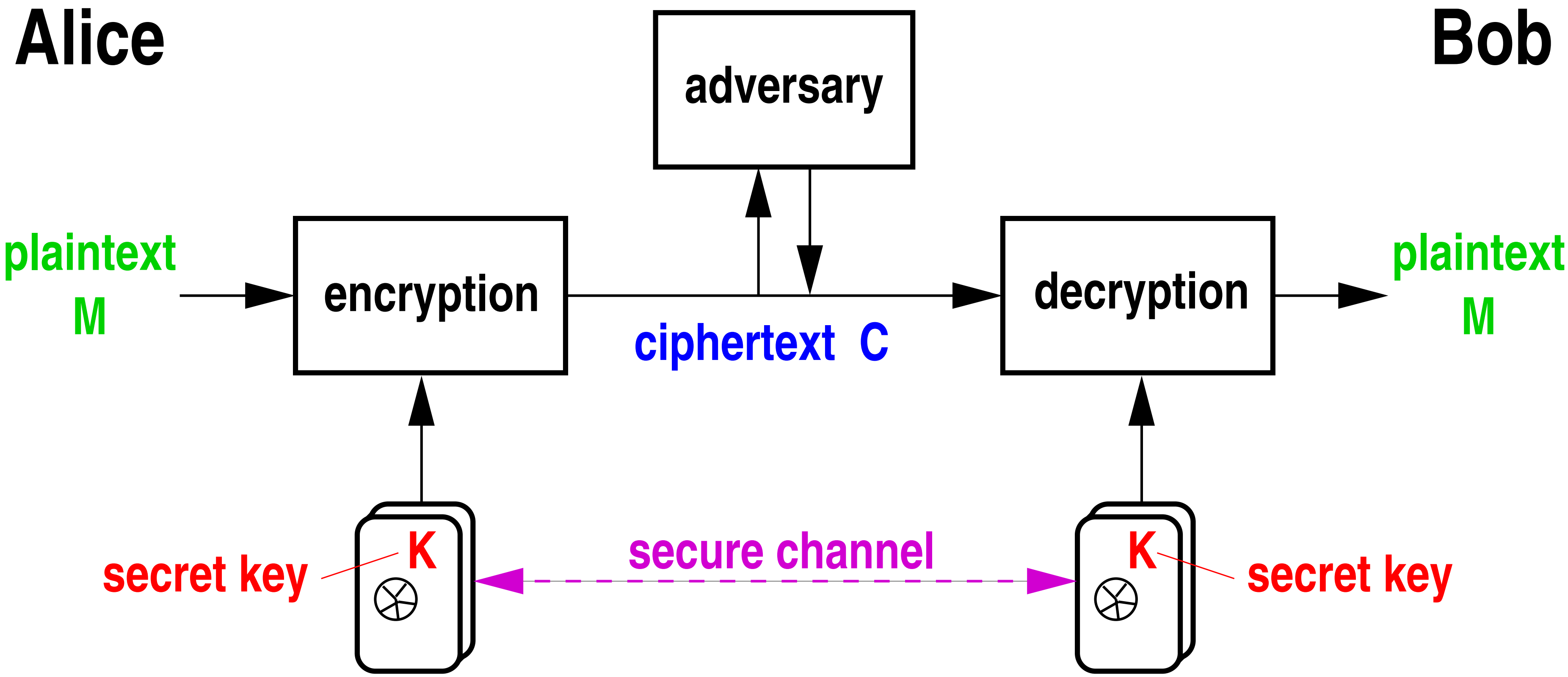
Key transport in •-calculus

$$A \bullet\!\!\longrightarrow\!\!\bullet\, B \quad \xrightarrow{\textbf{KT}} \quad A \bullet\!\!=\!\!=\!\!\bullet\, B$$

$$A \longrightarrow\!\!\bullet\, B \quad \xrightarrow{\textbf{KT}} \quad A =\!\!=\!\!\bullet\, B$$

Attention: $$A \bullet\!\!\longrightarrow B \quad \xrightarrow{\cancel{\textbf{KT}}} \quad A \bullet\!\!=\!\!= B$$

Symmetric cryptosystem



Symmetric cryptosystem in $\bullet$-calculus

$$\left.\begin{array}{l} A \Longrightarrow\!\!\bullet\ B \\ A \longrightarrow B \end{array}\right\} \xrightarrow{\text{SYM}} A \longrightarrow\!\!\bullet\ B$$

Symmetric cryptosystem in $\bullet$-calculus

$$\left.\begin{array}{l} A \;=\!\!=\!\!\bullet\; B \\ A \longrightarrow B \end{array}\right\} \xrightarrow{\textbf{SYM}} A \longrightarrow\!\!\bullet\; B$$

$$\left.\begin{array}{l} A \;=\!\!=\!\!\bullet\; B \\ A \;\bullet\!\!\longrightarrow B \end{array}\right\} \xrightarrow{\textbf{SYM}} A \;\bullet\!\!\longrightarrow\!\!\bullet\; B$$

Message authentication in •-calculus

$$\left.\begin{array}{l} A \;\bullet\!\!=\!\!=\!\!=\; B \\ A \longrightarrow B \end{array}\right\} \xrightarrow{\textbf{MAC}} A \bullet\!\!\longrightarrow B$$

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Note: Conservation law of •-calculus.

Combining Encryption and MAC

Goal: $\left.\begin{array}{c} A \;\bullet\!\!\!=\!\!=\!\!=\!\!\!\bullet\; B \\ A \longrightarrow B \end{array}\right\} \xrightarrow{???} A \;\bullet\!\!\!\longrightarrow\!\!\!\bullet\; B$

Combining Encryption and MAC

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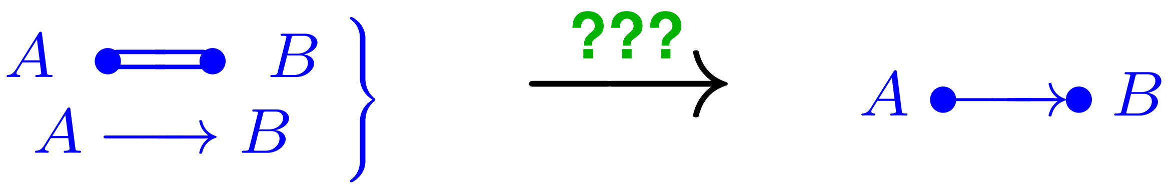
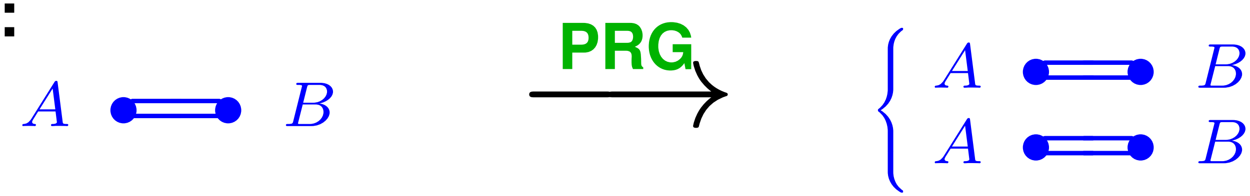


Key expansion:



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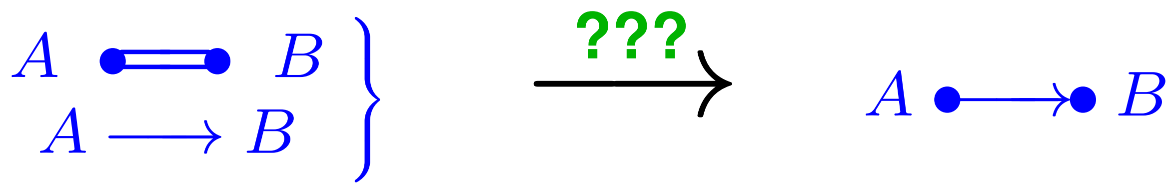

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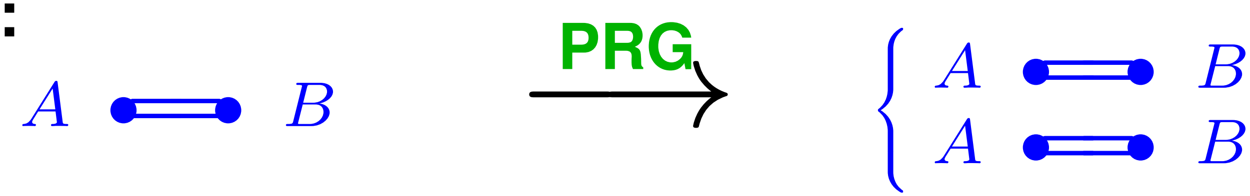
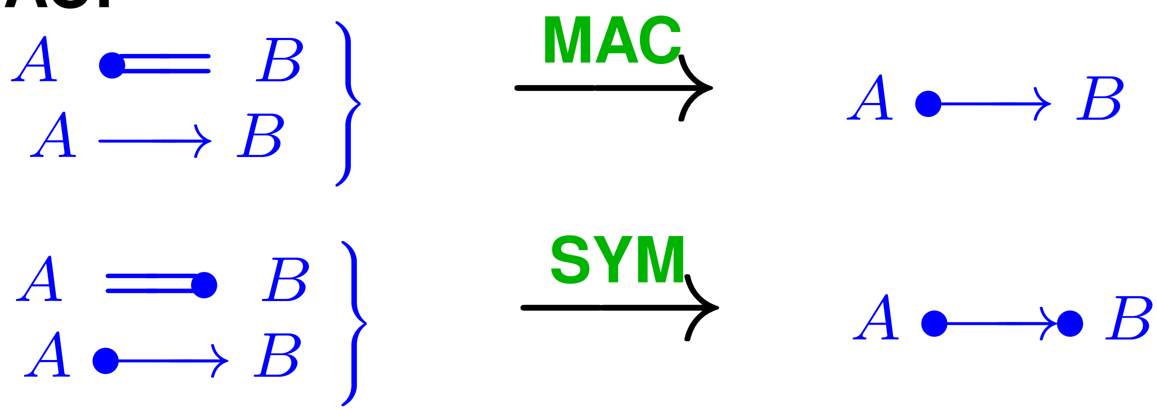


Encrypt-then-MAC:



Combining Encryption and MAC

Goal:

$$\left.\begin{array}{l} A \;\bullet\!\!=\!\!=\!\!\bullet\; B \\ A \longrightarrow B \end{array}\right\} \xrightarrow{???} A \;\bullet\!\!\longrightarrow\!\!\bullet\; B$$

Key expansion:

$$A \;\bullet\!\!=\!\!=\!\!\bullet\; B \xrightarrow{PRG} \left\{\begin{array}{l} A \;\bullet\!\!=\!\!=\!\!\bullet\; B \\ A \;\bullet\!\!=\!\!=\!\!\bullet\; B \end{array}\right.$$

Encrypt-then-MAC:

$$\left.\begin{array}{l} A \;\bullet\!\!=\!\!=\; B \\ A \longrightarrow B \end{array}\right\} \xrightarrow{MAC} A \;\bullet\!\!\longrightarrow B$$

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MAC-then-encrypt:

$$\left.\begin{array}{l} A \;=\!\!=\!\!\bullet\; B \\ A \longrightarrow B \end{array}\right\} \xrightarrow{SYM} A \longrightarrow\!\!\bullet\; B$$

$$\left.\begin{array}{l} A \;\bullet\!\!=\!\!=\; B \\ A \longrightarrow\!\!\bullet\; B \end{array}\right\} \xrightarrow{MAC} A \;\bullet\!\!\longrightarrow\!\!\bullet\; B$$

Combining Encryption and MAC

Goal:

$$\left.\begin{array}{l} A \;\bullet\!\!=\!=\!\!\bullet\; B \\ A \longrightarrow B \end{array}\right\} \xrightarrow{\;???\;} A \;\bullet\!\!\longrightarrow\!\!\bullet\; B$$

Key expansion:

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Encrypt-then-MAC:

$$\left.\begin{array}{l} A \;\bullet\!\!=\!=\; B \\ \ldots \end{array}\right\} \xrightarrow{\;\text{MAC}\;} A \;\bullet\!\!\longrightarrow B$$

Applies to computational and inform.-th. security.

$$\left.\begin{array}{l} \ldots \\ A \;\bullet\!\!\longrightarrow B \end{array}\right\} \longrightarrow A \;\bullet\!\!\longrightarrow\!\!\bullet\; B$$

MAC-then-encrypt:

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Combining Encryption and MAC

Goal: $\left.\begin{array}{c} A \;\bullet\!\!=\!\!=\!\!=\!\!\bullet\; B \\ A \longrightarrow B \end{array}\right\} \xrightarrow{\text{???}} A \bullet\!\!\longrightarrow\!\!\bullet B$

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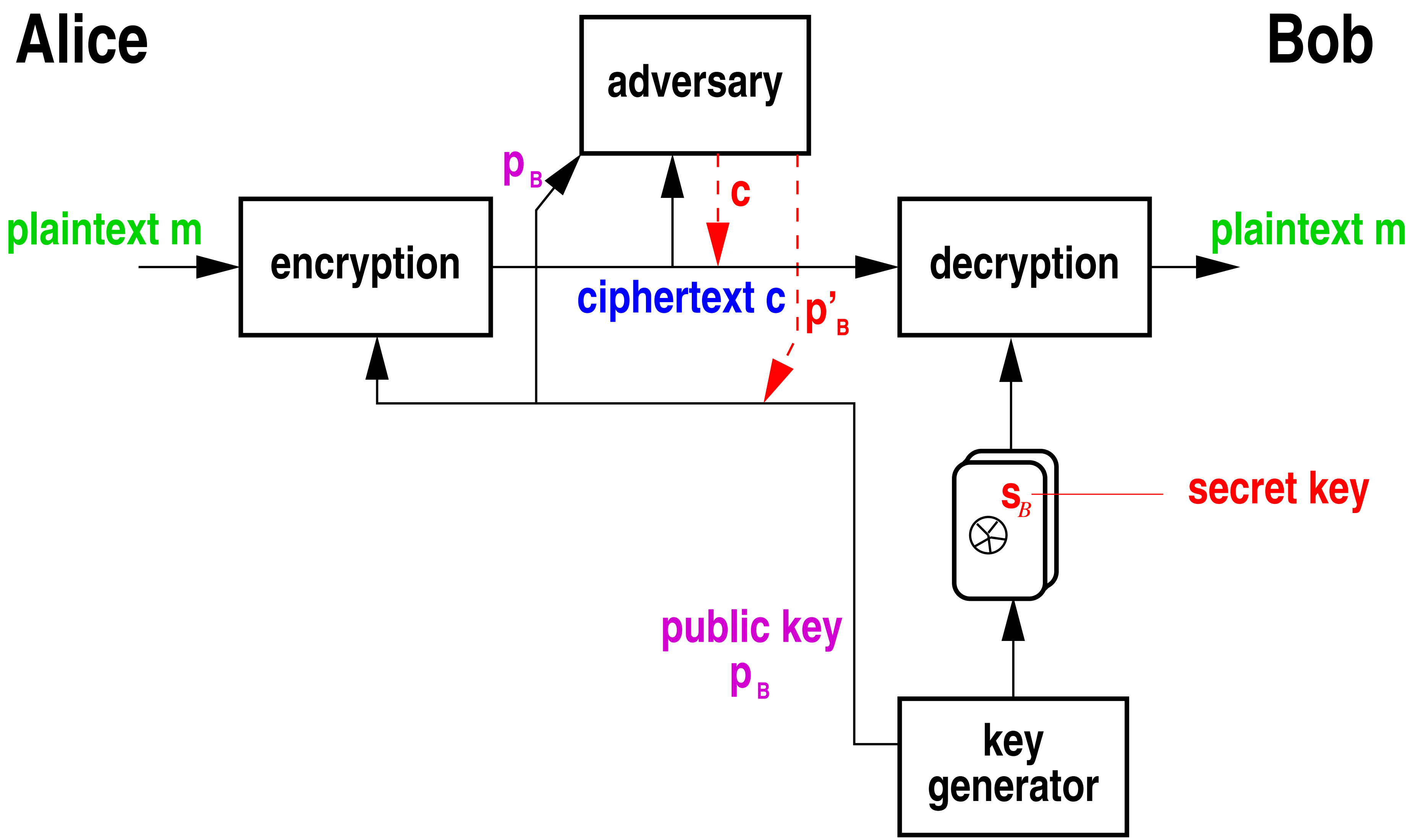
$\left.\begin{array}{c} A \;\bullet\!\!=\!\!=\!\!=\; B \\ A \longrightarrow\!\!\bullet B \end{array}\right\} \xrightarrow{\text{MAC}} A \bullet\!\!\longrightarrow\!\!\bullet B$

Public-key cryptosystem



Public-key cryptosystem in $\bullet$-calculus

$$\left.\begin{array}{c} A \bullet\!\!\longrightarrow B \\ A \longleftarrow B \end{array}\right\} \xrightarrow{\textbf{PKC}} A \bullet\!\!\longleftarrow B$$

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Key agreement in •-calculus

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$$\left.\begin{array}{l} A \bullet\!\!\longrightarrow B \\ A \longleftarrow\!\!\bullet B \\ A \relbar Q \!\!\rightarrow B \end{array}\right\} \xrightarrow{\textbf{QKD}} A \bullet\!\!=\!\!\bullet B$$

Key agreement in $\bullet$-calculus

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$$\left.\begin{array}{l} A \bullet\!\!\longrightarrow B \\ A \longleftarrow\!\!\bullet B \\ A \,-\!Q\!\!\longrightarrow B \end{array}\right\} \xrightarrow{\textbf{QKD}} A \bullet\!\!=\!\!=\!\!\bullet B$$

Note: Conservation law of $\bullet$-calculus.

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Note: Conservation law of •-calculus.

Digital signature scheme in •-calculus

$$\left.\begin{array}{c} A \bullet\!\!\longrightarrow B \\ A \longrightarrow B \end{array}\right\} \xrightarrow{\textbf{DSS}} A \bullet\!\!\longrightarrow B$$

Information-theoretic authentication

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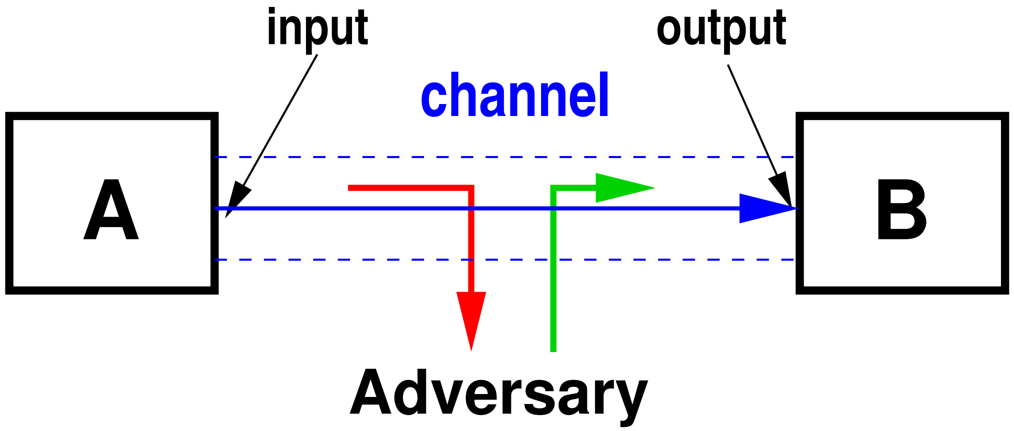
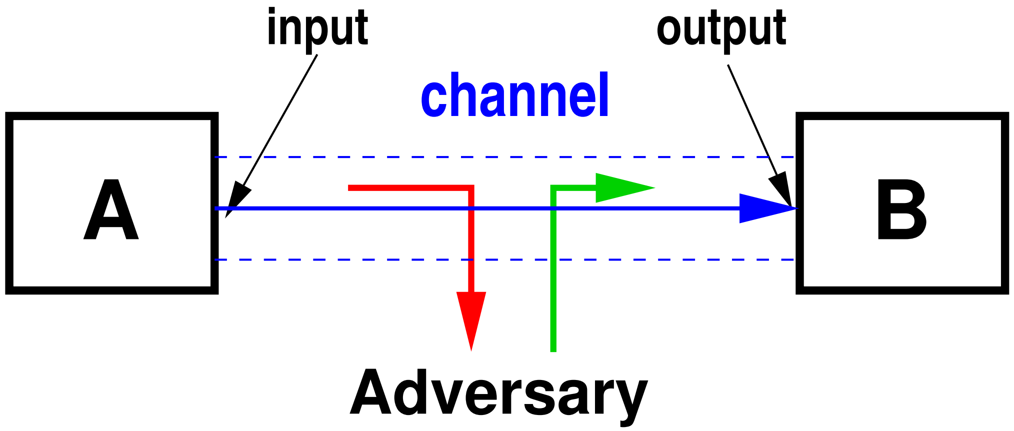


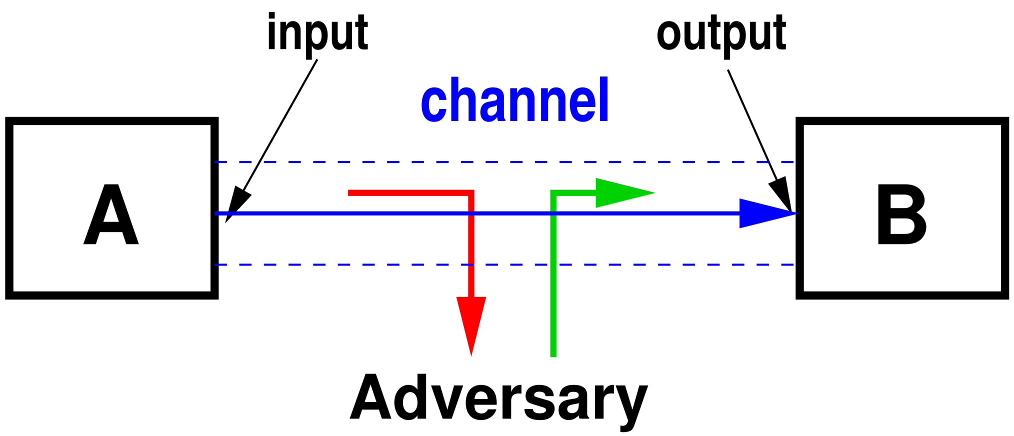
Information-theoretic authentication



Impersonation attack: The adversary sends a fraudulent message **before** observing the real message.

Success probability: P_I

Information-theoretic authentication



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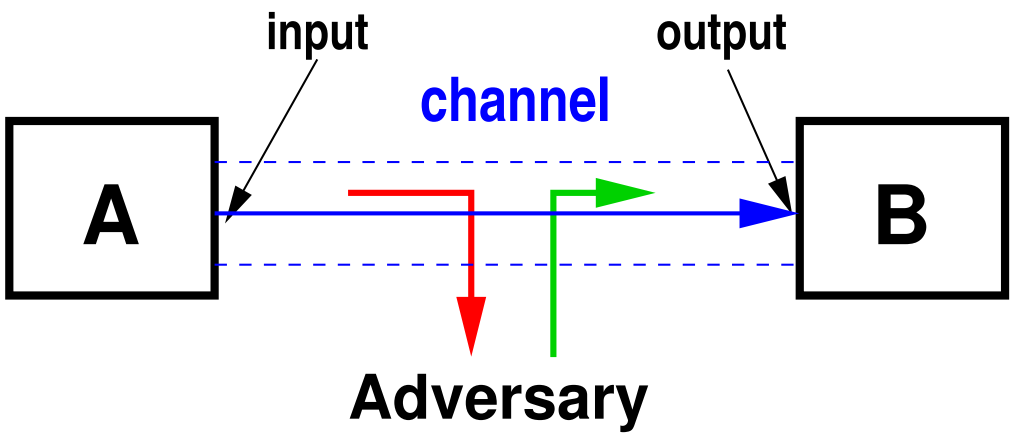
Note: $P_I \geq |\mathcal{M}|/|\mathcal{C}|$.

Information-theoretic authentication



Impersonation attack: The adversary sends a fraudulent message **before** observing the real message.

Success probability: P_I

Note: $P_I \geq |\mathcal{M}|/|\mathcal{C}|$.

Substitution attack: The adversary sends a fraudulent message **after** observing a correctly auth. message.

Success probability: P_S

Authenticating an ℓ-bit message with a k-bit key

Authenticating an ℓ-bit message with a k-bit key

$$\left.\begin{array}{ccc} A & \bullet\!\!\overset{k}{=\!=\!=}\!\!\bullet & B \\ A & \xrightarrow{\ \ell\ } & B \end{array}\right\} \xrightarrow{\text{ITA}} A \bullet\!\!\xrightarrow{\ \ell\ } B$$

Example 1: $M \in \{0,1\}, \quad C = M||K$

Authenticating an ℓ-bit message with a k-bit key

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Example 1: $M \in \{0,1\}$, $\quad C = M||K \quad P_I = 2^{-k}, \quad P_S = 1$

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Example 2: $M \in \{0,1\}$

$$K = K_1||K_0 \text{ with } K_0, K_1 \in \{0,1\}^{k/2}$$

$$C = \begin{cases} 0||K_0 & \text{if } M = 0 \\ 1||K_1 & \text{if } M = 1 \end{cases}$$

Authenticating an ℓ-bit message with a k-bit key

$$\left.\begin{array}{ccc} A & \overset{k}{\bullet\!=\!=\!=\!\bullet} & B \\ A & \xrightarrow{\ell} & B \end{array}\right\} \xrightarrow{\text{ITA}} \quad A \overset{\ell}{\bullet\!\!\longrightarrow} B$$

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$$P_I = 2^{-k/2}, \quad P_S = 2^{-k/2}$$

Authenticating an ℓ-bit message with a k-bit key

$$\left.\begin{array}{ccc} A & \overset{k}{\bullet\!=\!=\!\bullet} & B \\ A & \xrightarrow{\ell} & B \end{array}\right\} \quad \xrightarrow{\text{ITA}} \quad A \bullet\!\xrightarrow{\ell} B$$

Example 1: $M \in \{0,1\}, \quad C = M||K \quad P_I = 2^{-k}, \quad P_S = 1$

Example 2: $M \in \{0,1,2\}$

$$K = K_1||K_0 \text{ with } K_0, K_1 \in \{0,1\}^{k/2}$$

$$C = \begin{cases} 0||K_0 & \text{if } M = 0 \\ 1||K_1 & \text{if } M = 1 \\ 2||K_0 \oplus K_1 & \text{if } M = 2 \end{cases}$$

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Exampl $P_S = 1$

Q: Is a lower cheating probability possible?

Exampl

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Exampl … $P_S = 1$

Q: Is a lower cheating probability possible?

Exampl …

Q: What about longer messages?

$$C = \begin{cases} 1 \| K_1 & \text{if } M = 1 \\ 2 \| K_0 \oplus K_1 & \text{if } M = 2 \end{cases}$$

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Lower bounds on the cheating probability

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Lower bounds on the cheating probability

Theorem: For every authentication system we have

$$P_I \geq 2^{-I(C;K)}$$

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Lower bounds on the cheating probability

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\max(P_I, P_S) &\geq 2^{-H(K)/2} = 2^{-k/2}
\end{aligned}
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$$\boxed{k \geq 2s}$$

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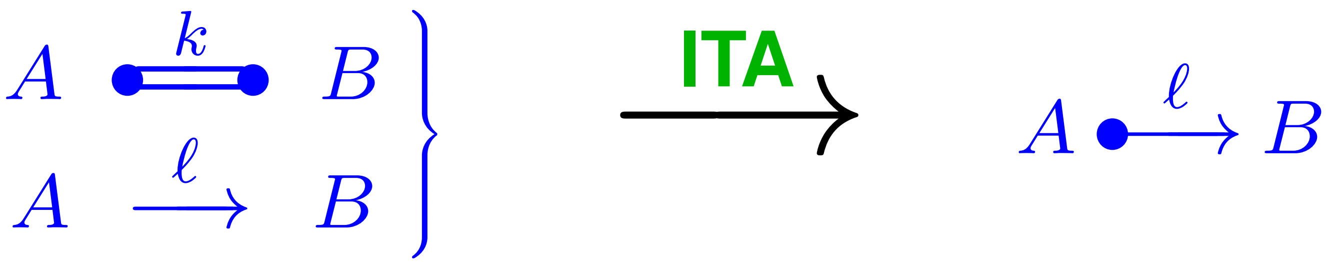
Authenticating an ℓ-bit message with a k-bit key



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Block length n, field $F = GF(2^n)$,

$m = [m_{b-1}, \ldots, m_1, m_0]$, $\qquad \ell = bn$

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Q: Trade-off between ℓ, k, s ?

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Block length n, field $F = G$

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This is essentially optimal!

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$m = [m_{b-1}, \ldots, m_1, m_0]$,

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Authenti $+ K_0$

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This is essentially optimal!

Q: Can we nevertheless do better?

Theorem: $P_I = P_S = b \cdot 2^{-n}$; $s = \frac{k}{2} - \log(2\ell/k)$

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Authenticating an ℓ-bit message by auth. t bits

$$\left.\begin{array}{l} A \xrightarrow{\ell} B \\ A \bullet\!\xrightarrow{t} B \end{array}\right\} \xrightarrow{\textbf{A-Ampl}} A \bullet\!\xrightarrow{\ell} B$$

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Protocol (A-Ampl): Send m over $A \xrightarrow{\ell} B$, then $R||p_m(R)$ over $A \bullet\!\!\xrightarrow{t} B$, for a random R.

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Q: What does all of this really mean?

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Q: What does all of this really mean? (e.g. for QKD?)

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Security definitions in classical cryptography

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Definition: A **public-key cryptosystem (PKC)** is a triple of polynomial-time algorithms (PPT) with security parameter k:

1. **KeyGen:** input: k; output: a secret key s, a public key p.
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Security: A PKC is **IND-CPA secure** if no probabilistic polynomial time-bounded adversary A can win the following game with probability non-negligibly greater than 1/2:

1. p is generated with **KeyGen**, and given to A.
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A1: Constructive cryptography

W on?

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pol

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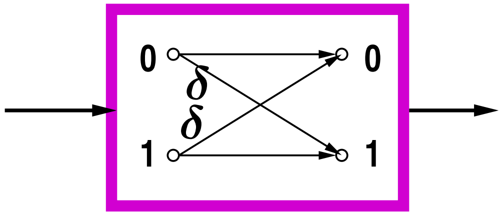
Shannon's channel coding theorem

Shannon's channel coding theorem

n−bit noisy channel



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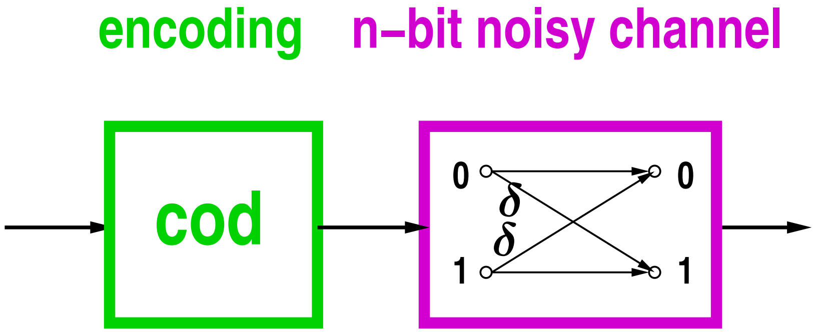

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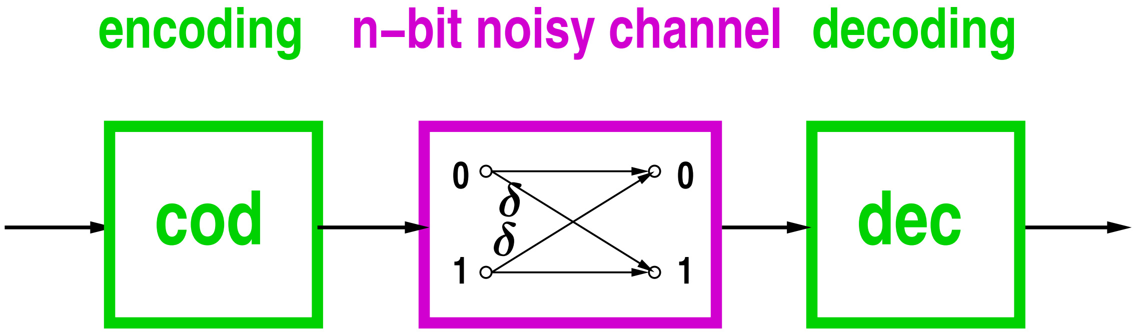

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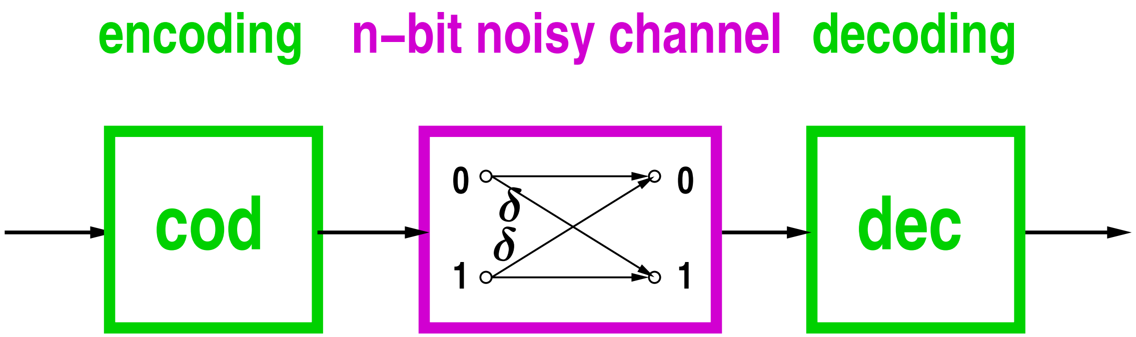

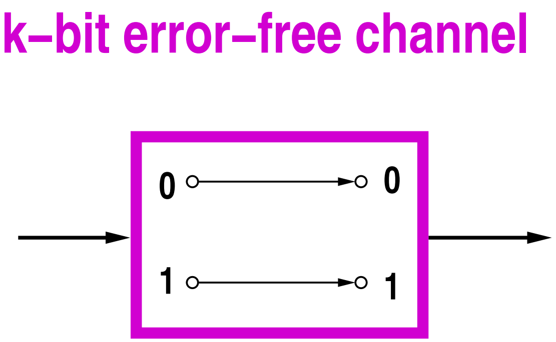

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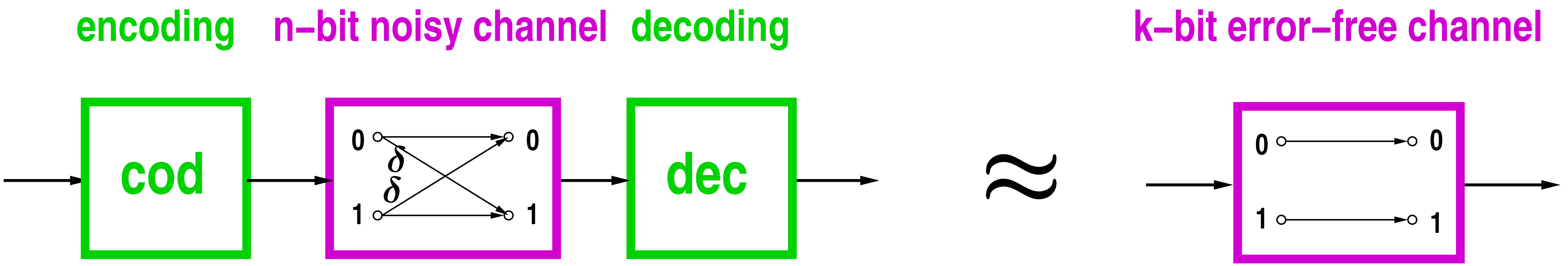

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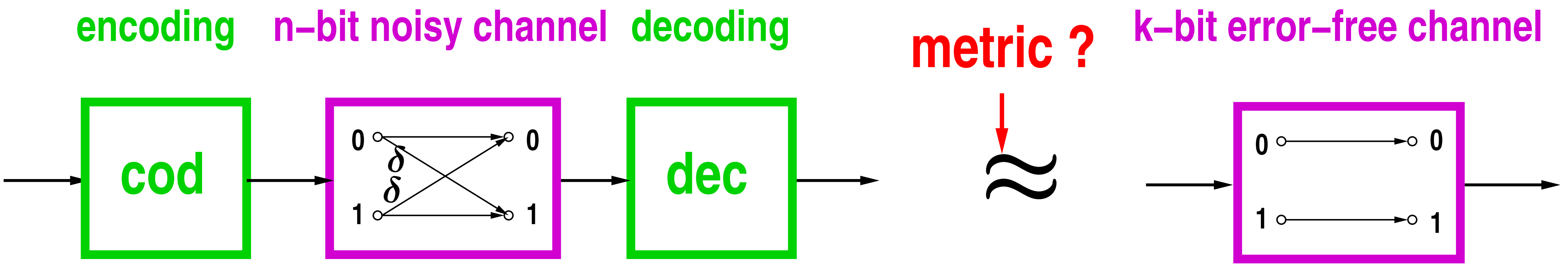

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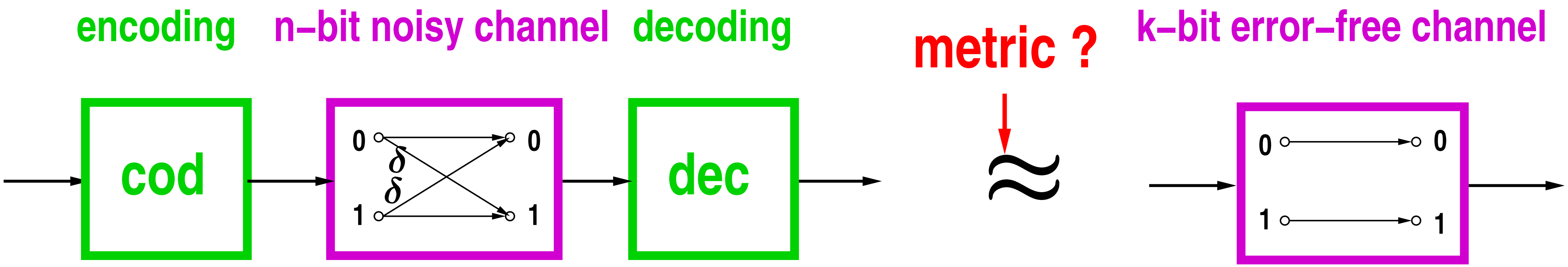

cod $\quad \mathrm{BSC}_\delta^n \quad$ dec $\quad \approx \quad \mathrm{BSC}_0^k$

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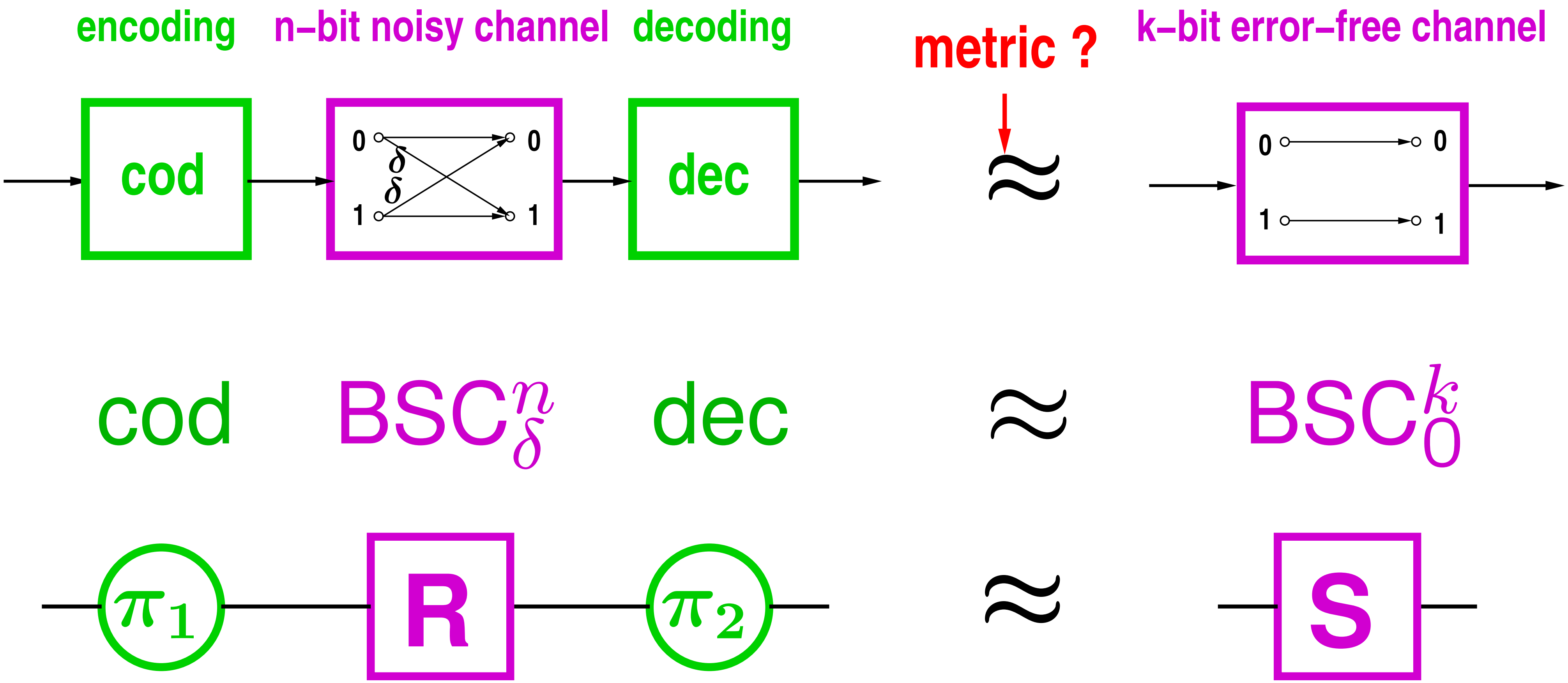

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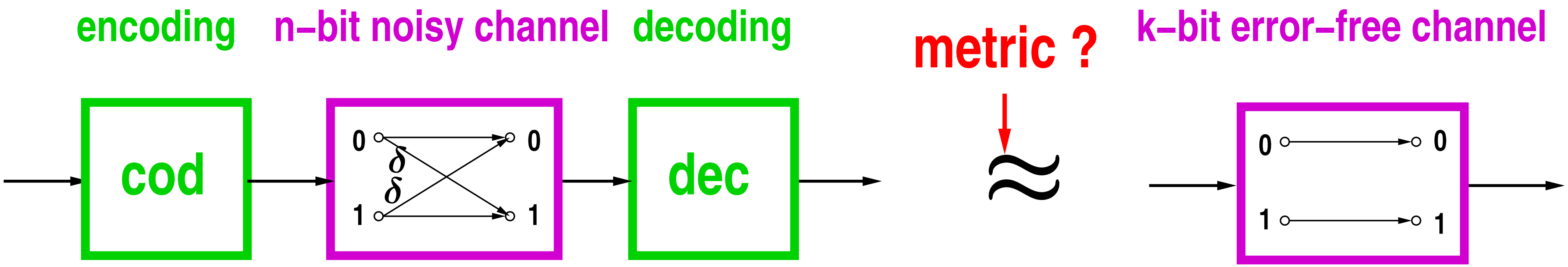

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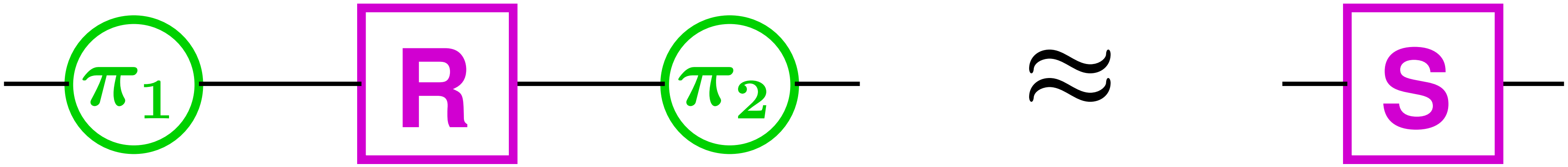

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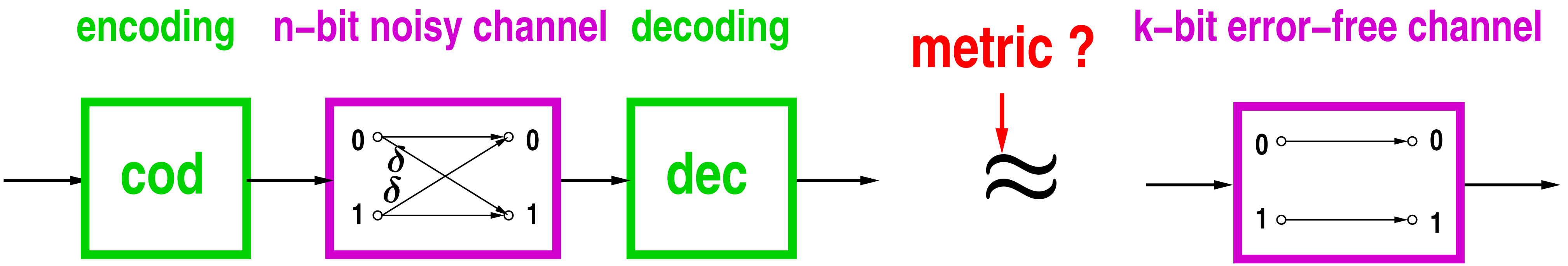

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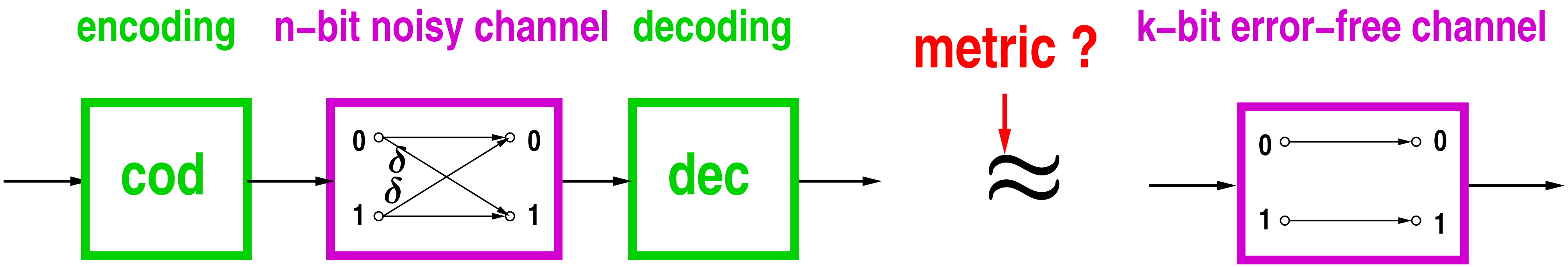

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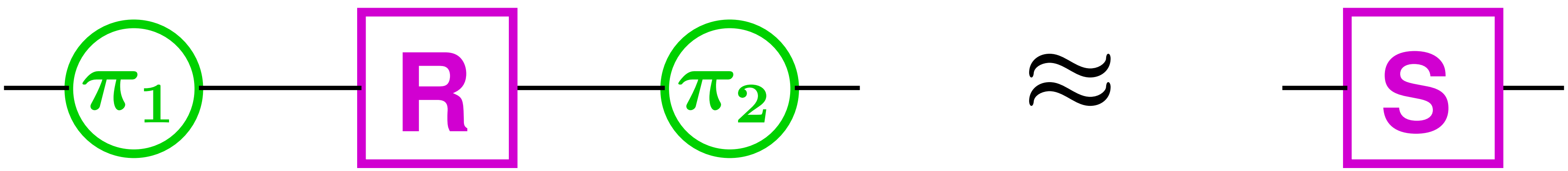

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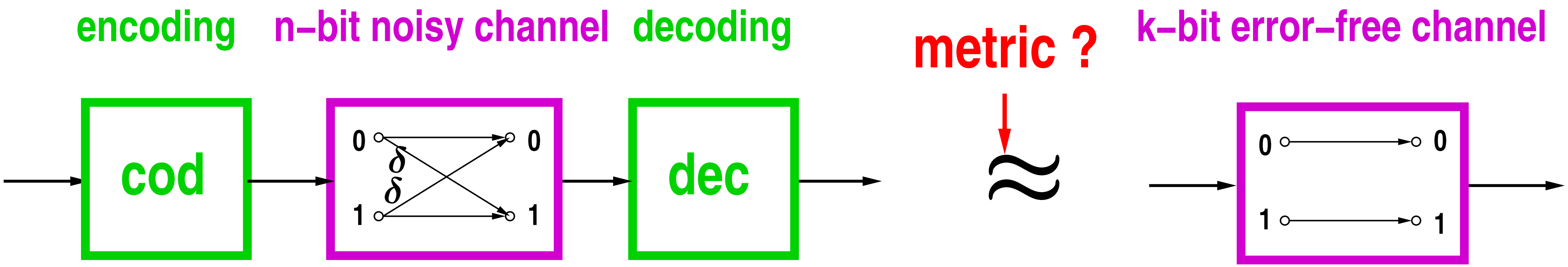

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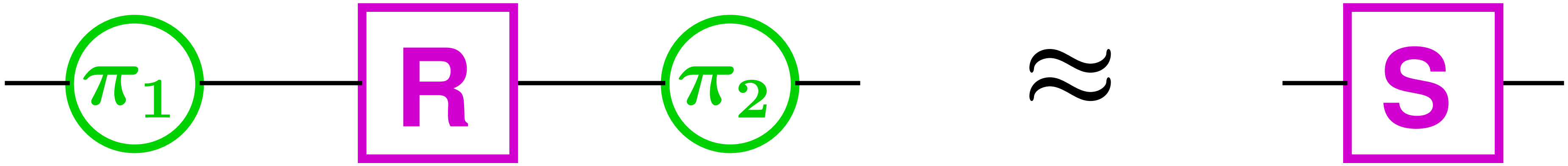

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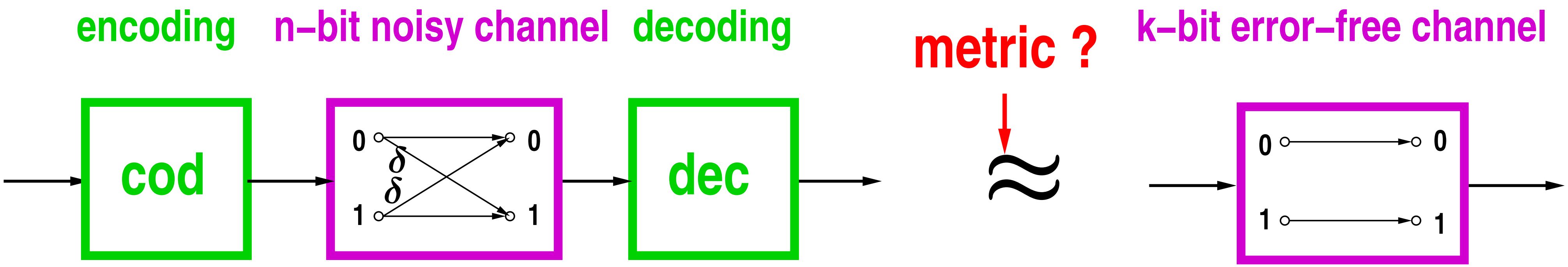

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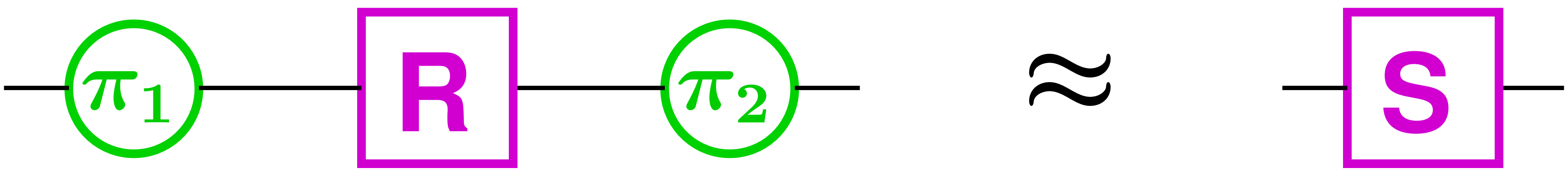

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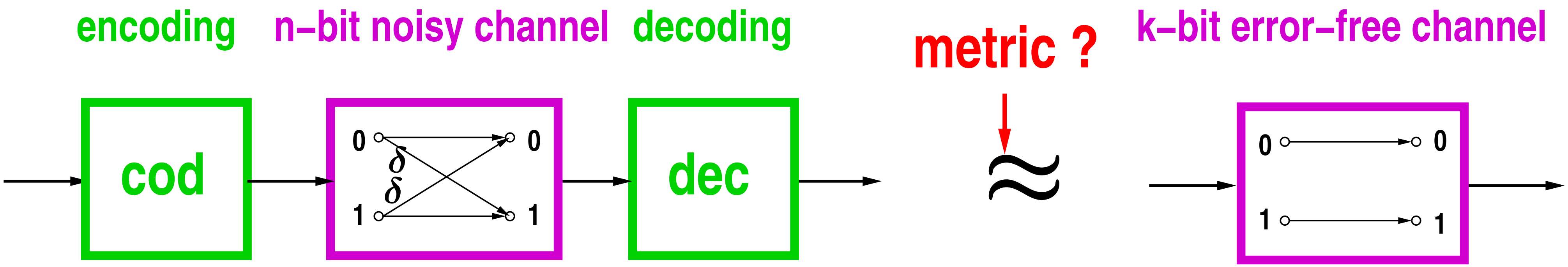

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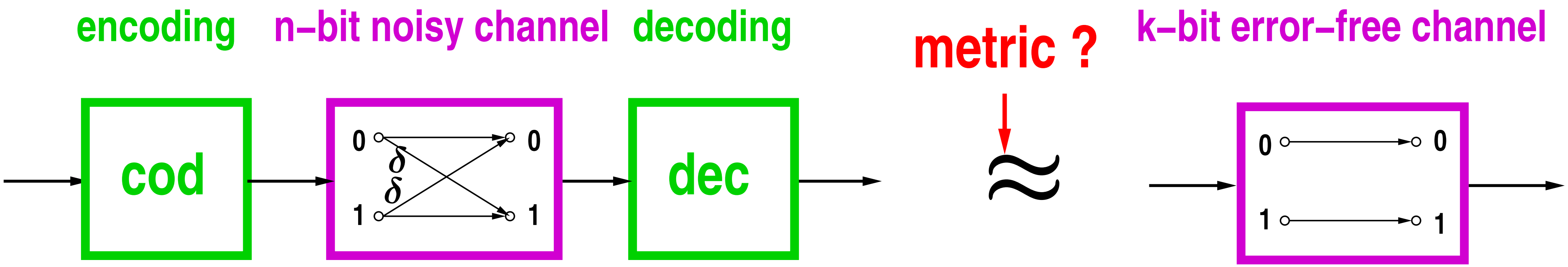

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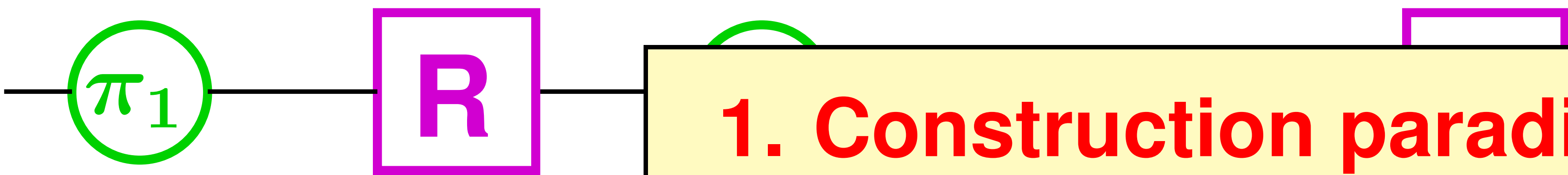

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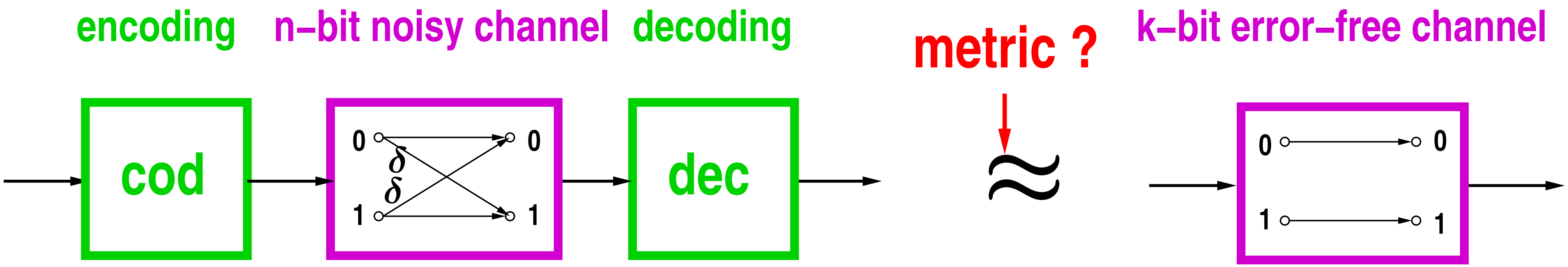

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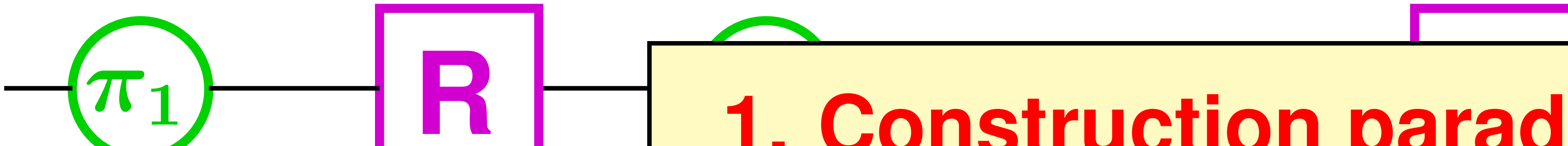

1. Construction paradigm

2. Abstract system algebra

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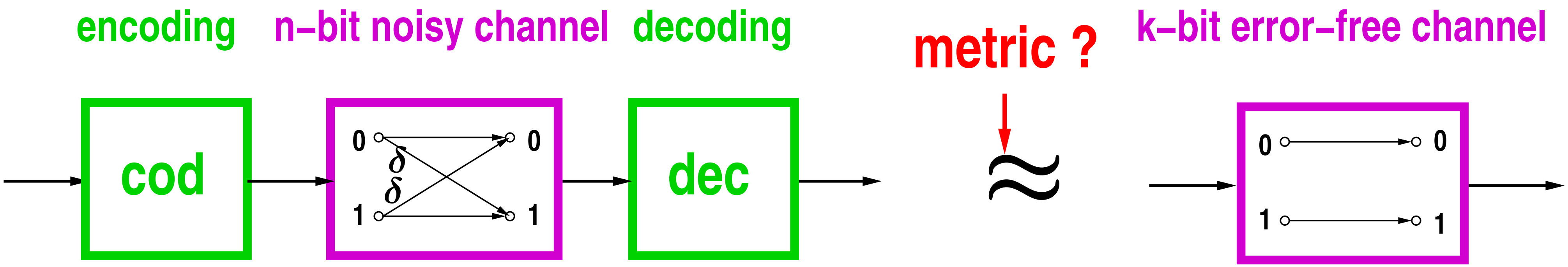

cod BSC

3. Constructive cryptography

π_1 R

1. Construction paradigm

2. Abstract system algebra

on:

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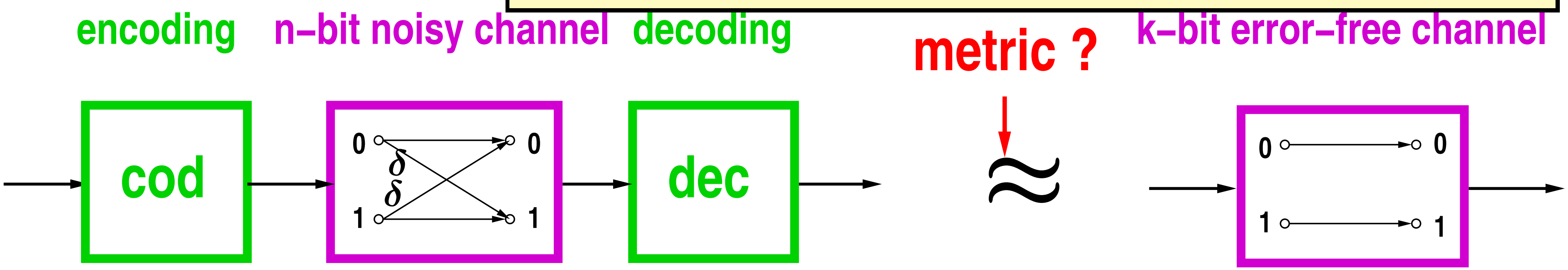
Shannon's channel coding theorem

4. Discrete systems, metric



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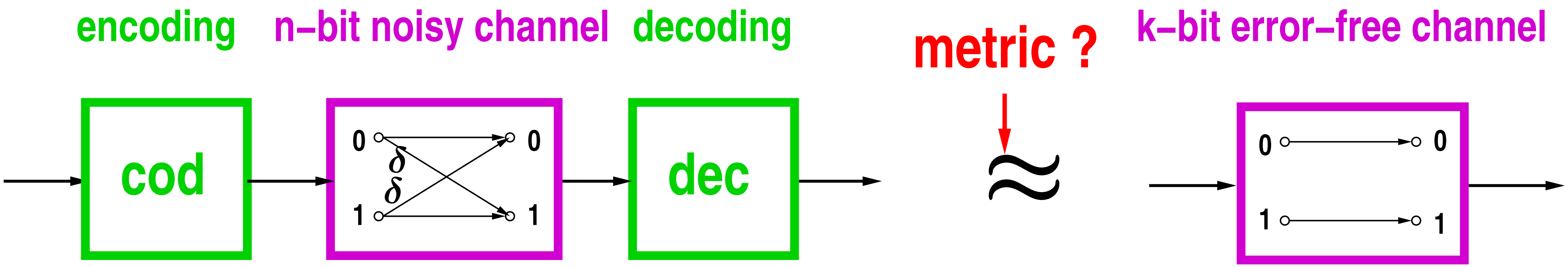

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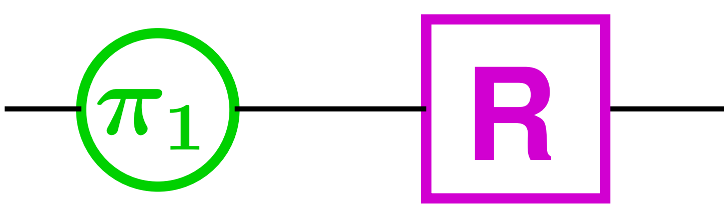

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The construction paradigm

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$$\mathbf{R} \xrightarrow{\alpha} \mathbf{S}$$

Construct an object **S** from another object **R** via construction α.

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Examples:

$$\mathrm{BSC}_{\delta}^{n} \xrightarrow{\text{(cod,dec)},\epsilon} \mathrm{BSC}_{0}^{k}$$

The construction paradigm

$$\mathbf{R} \xrightarrow{\alpha} \mathbf{S}$$

Construct an object **S** from another object **R** via construction α.

Examples:

A (k, m)-pseudo-random generator (PRG) constructs a uniform m-bit string from a uniform k-bit string:

$$\mathrm{U}_k \xrightarrow{\text{PRG}} \mathrm{U}_m$$

The construction paradigm

$$\mathbf{R} \xrightarrow{\alpha} \mathbf{S}$$

Construct an object **S** from another object **R** via construction α.

Examples:

A key agreement protocol (KAP) constructs a shared secret n-bit key from ???:

$$??? \xrightarrow{\text{KAP}} \text{KEY}_n$$

The construction paradigm

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Construct an object **S** from another object **R** via construction α.

Examples:

A complexity-theoretic reduction constructs an efficient algorithm for problem P from an efficient algorithm for problem Q.

The construction paradigm

$$\mathbf{R} \xrightarrow{\alpha} \mathbf{S}$$

Construct an object **S** from another object **R** via construction α.

Formally: set of objects Ω,
constructor set $\langle \Gamma, \circ, \mathrm{id} \rangle$,
construction $\subseteq \Omega \times \Gamma \times \Omega$

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Definition: A construction is **composable** if

$$\mathbf{R} \xrightarrow{\alpha} \mathbf{S} \;\wedge\; \mathbf{S} \xrightarrow{\beta'} \mathbf{T} \;\Rightarrow\; \mathbf{R} \xrightarrow{\alpha \circ \beta} \mathbf{T}$$

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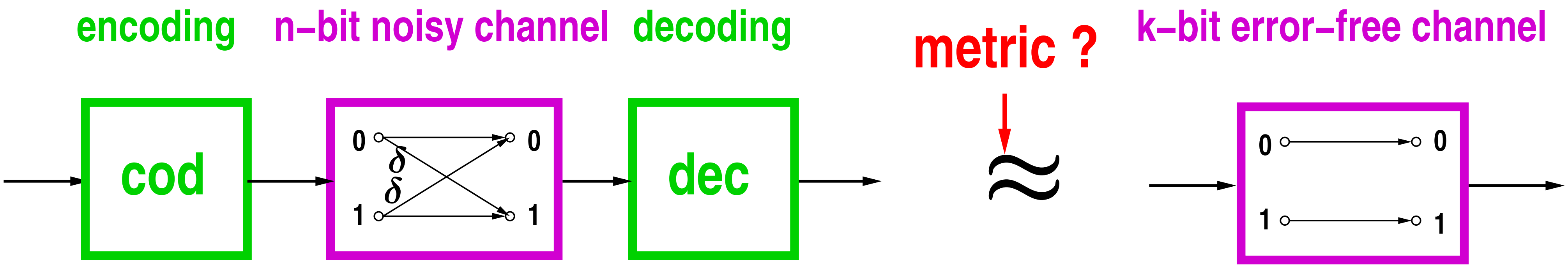

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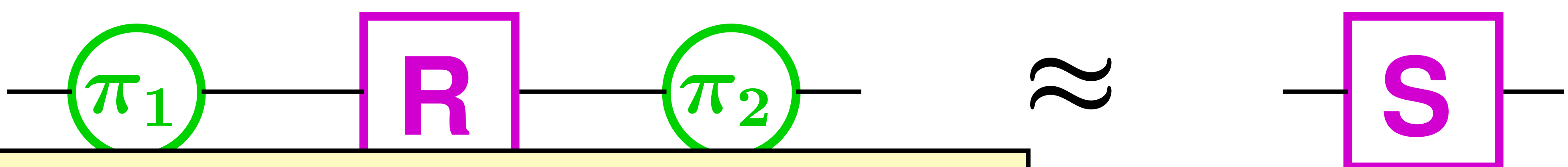

2. Abstract system algebra

on:

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A dilemma in computer science

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$\Rightarrow$ **enormous complexity, imprecise papers, ...**

A dilemma in computer science

Proposed paradigm shift in Computer Science:

Top-down abstraction

instead of

bottom-up definitions

security, ...

$\Rightarrow$ Turing machines, communication tapes, asymptotics, polynomial-time, ...

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A dilemma in computer science

Proposed paradigm shift in Computer Science:

Top-down abstraction

instead of

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security

Goals of abstraction:

- eliminate irrelevant details, minimality
- simpler definitions
- generality of results
- simpler proofs, elegance
- didactic suitability, better understanding

$\Rightarrow$

$\Rightarrow$ **e** ... **rs, ...**

Abstract system algebra $\langle \Phi, \Sigma \rangle$ [M-Renner11]

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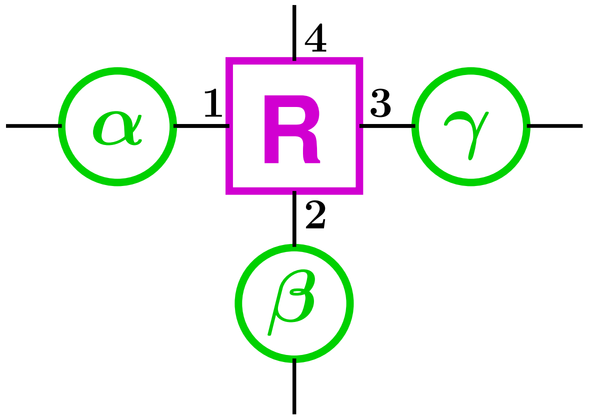
Resource set Φ for interface set $\mathcal{I}$ (e.g. $\mathcal{I} = \{1, 2, 3, 4\}$)

Converter set Σ



Algebraic laws:

- $\mathbf{R} || \mathbf{S} \in \Phi$ notation: $[\mathbf{R}, \mathbf{S}]$
- $\alpha^i \mathbf{R} \in \Phi$ for all $\mathbf{R} \in \Phi$, $\alpha \in \Sigma$, $i \in \mathcal{I}$
- $\alpha^i \beta^j \mathbf{R} = \beta^j \alpha^i \mathbf{R}$ for all $i \neq j$
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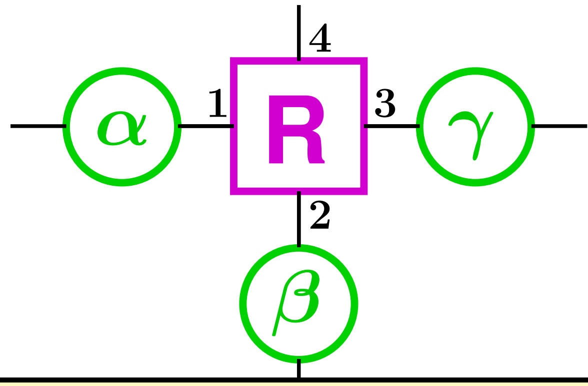

Pseudo-metric d on Φ:

Def.: **d** is **non-expanding** $\Longleftrightarrow$ **d**(α^i**R**,α^i**S**) $\leq$ **d**(**R**,**S**)

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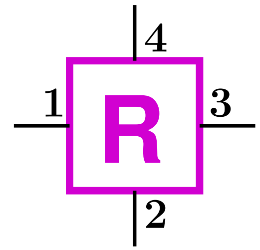

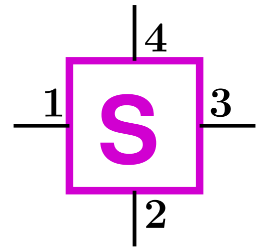

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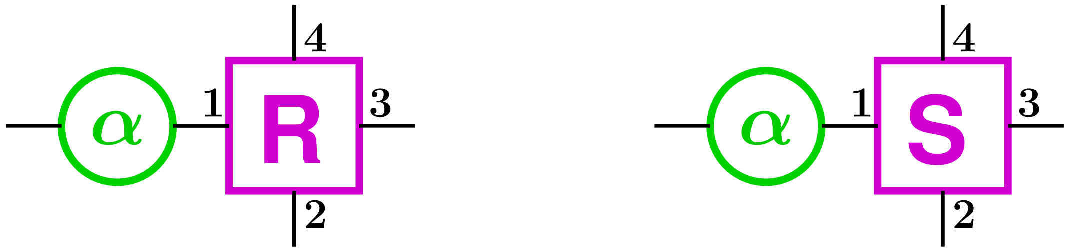

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Levels of abstraction

#	level	concepts treated at this level
0.	Constructions	composability, construction trees
1.	Abstract systems	composability proof
2.	Discrete systems	I/O bahavior, indistinguish. proofs
3.	System implem.	complexity, efficiency, asymptotics

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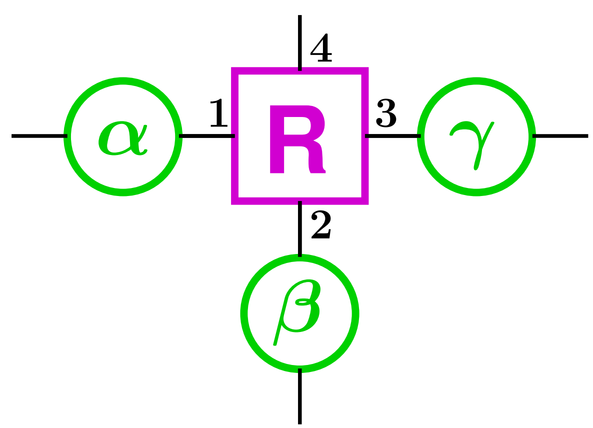

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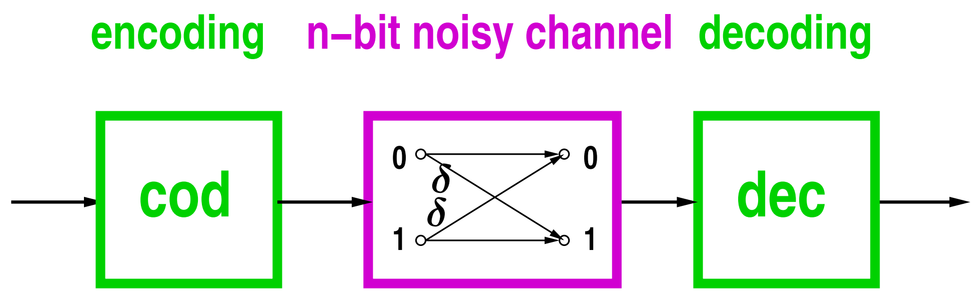

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system ENCRYPT
read x at outside interface
read k at inside interface
$c \leftarrow$ enc(x, k)
.....

Abstraction levels in algebra:

1. **Abstract group:** $\langle G, \star, e, (\cdot)^{-1} \rangle$
2. **Instantiations:** Integers, real number, elliptic curves
3. **Representations:** e.g. projective coordinates for ECs

1.	**Abstract systems**	composability proof
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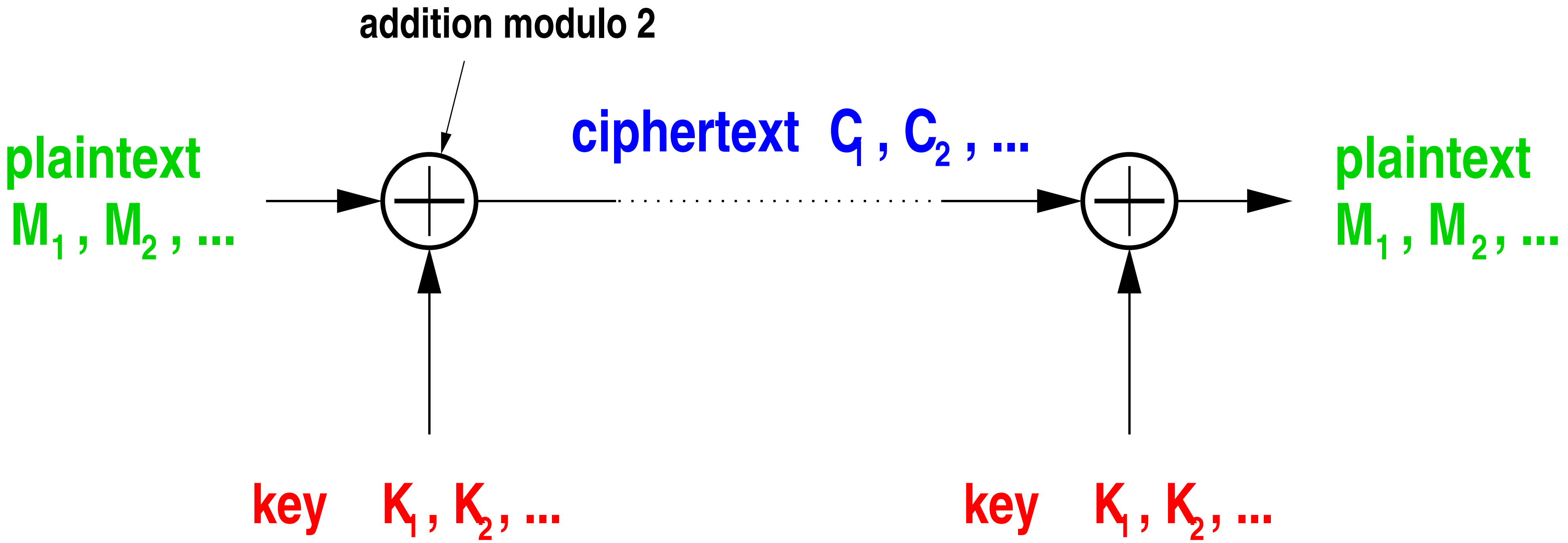
Constructive cryptography

Constructive cryptography

One-time pad:



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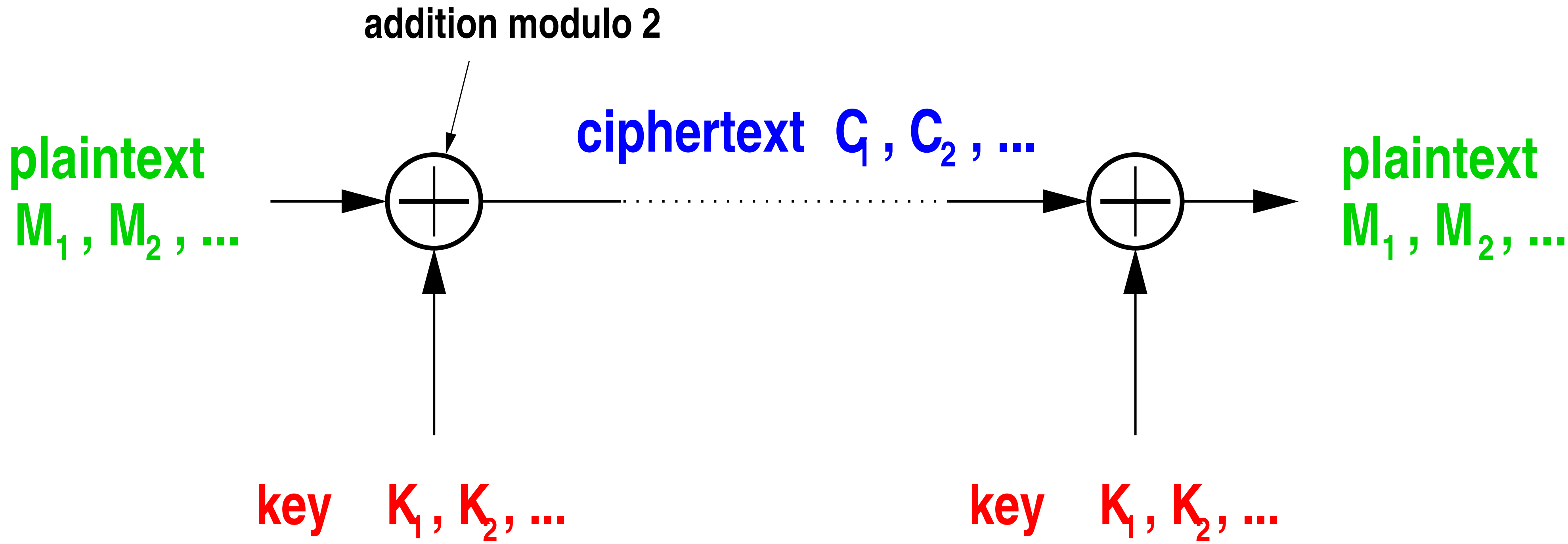

Security ?

Constructive cryptography

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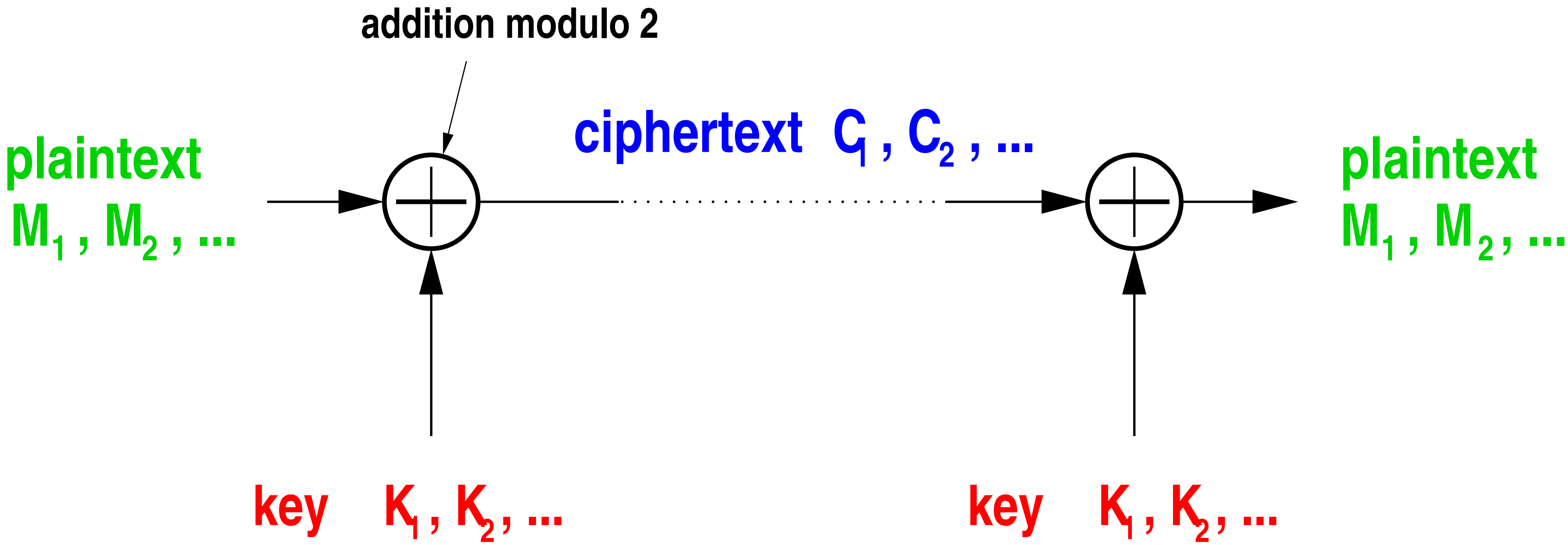

Security [SHANNON]: $\mathbf{I}(\mathbf{C},\mathbf{M}) = 0$ (perfect secrecy)

One-time pad in constructive cryptography

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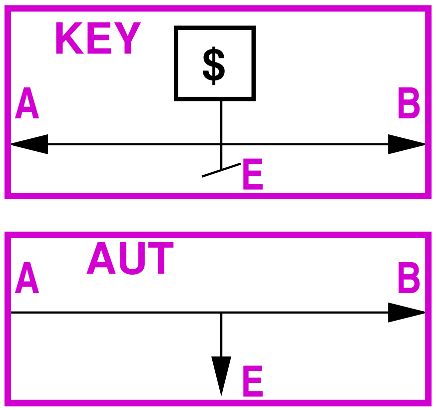

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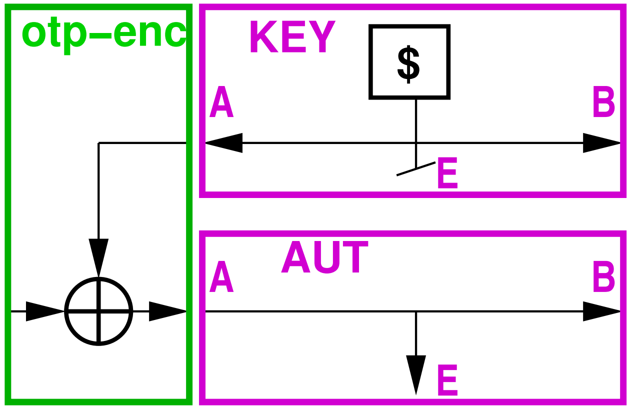

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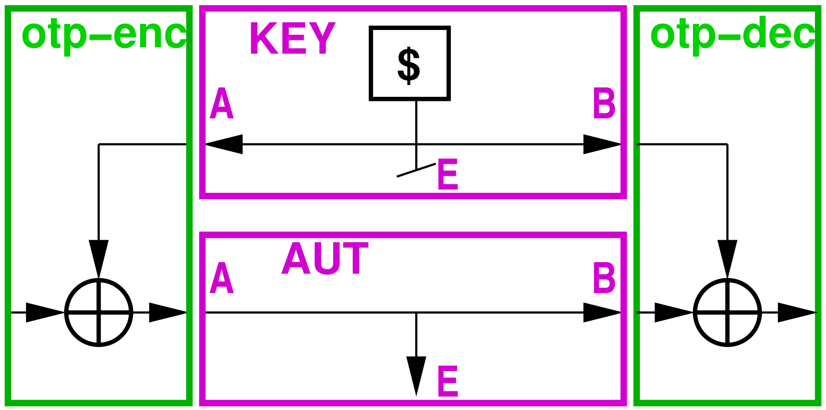

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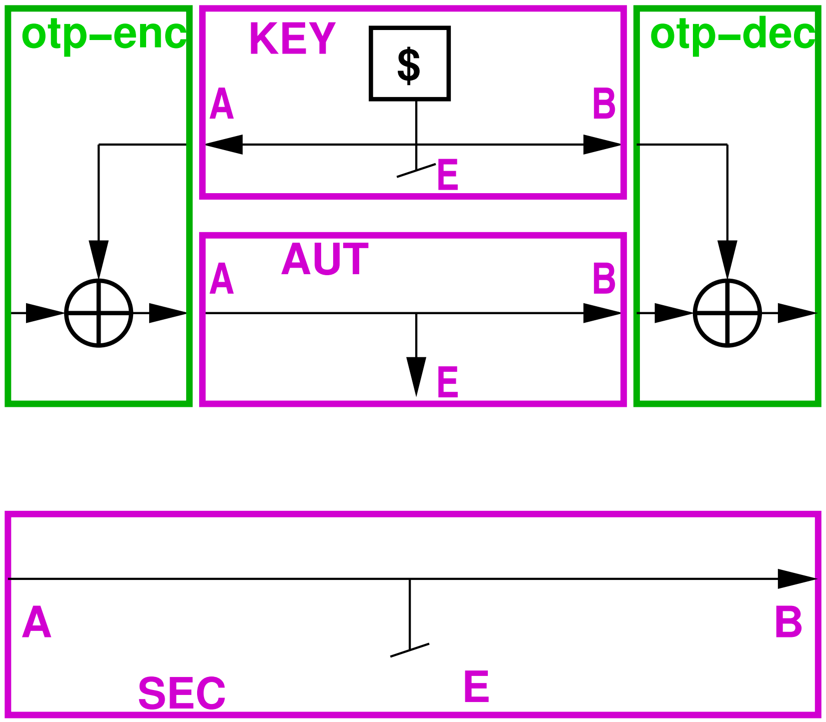

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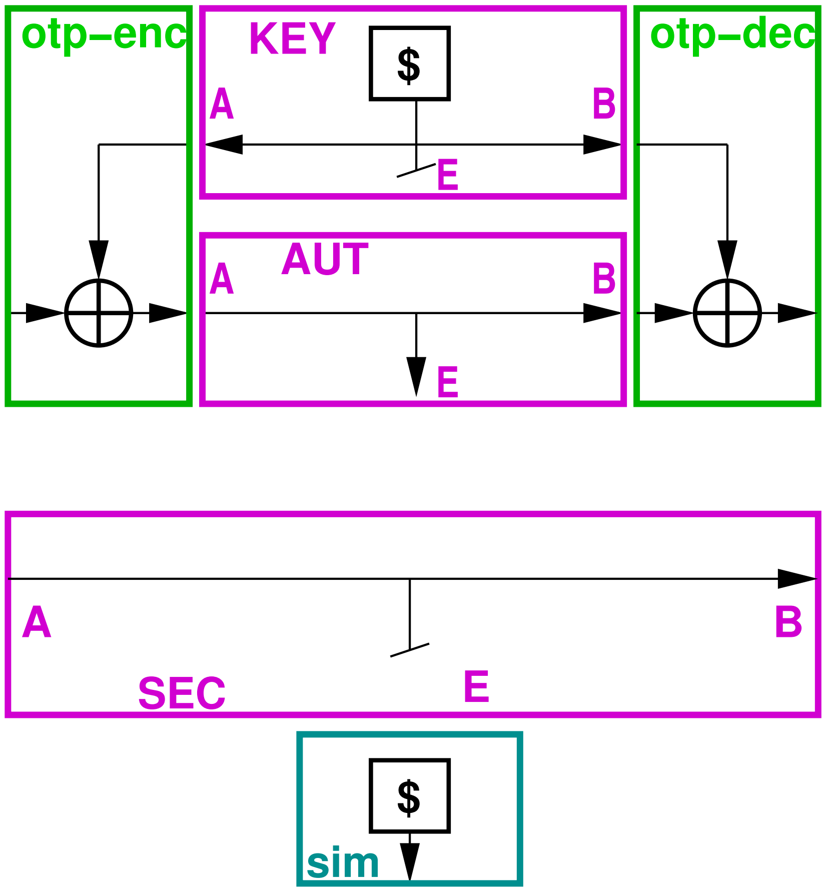

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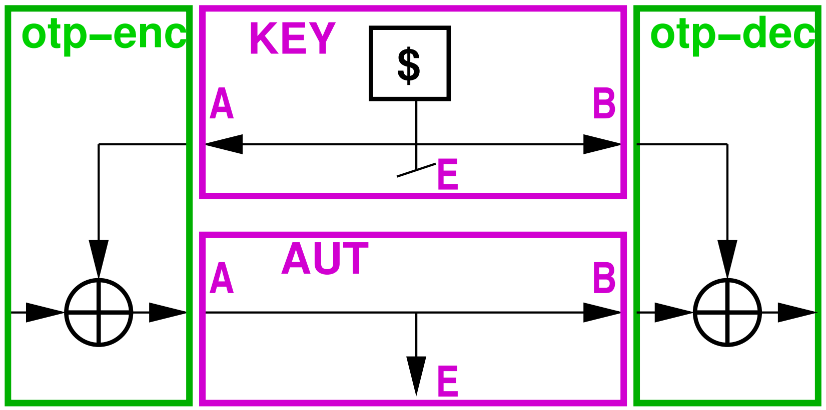

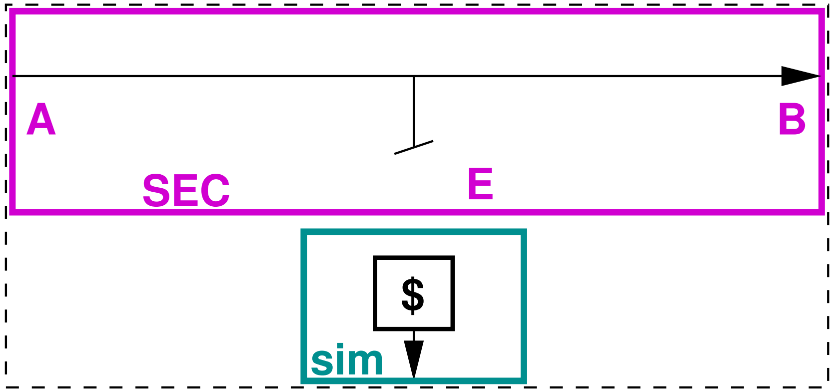

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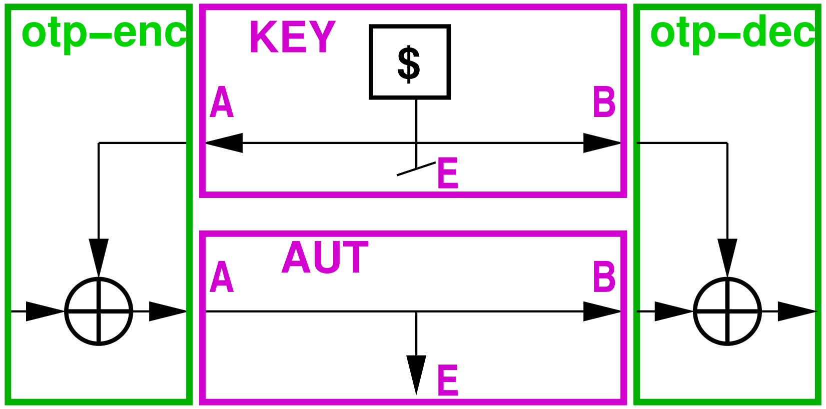

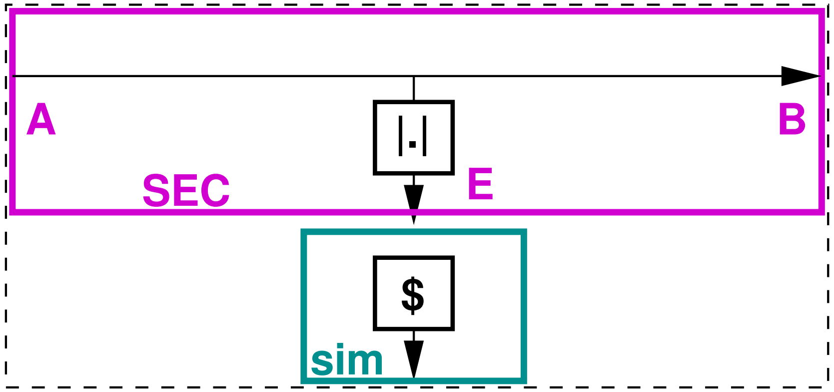

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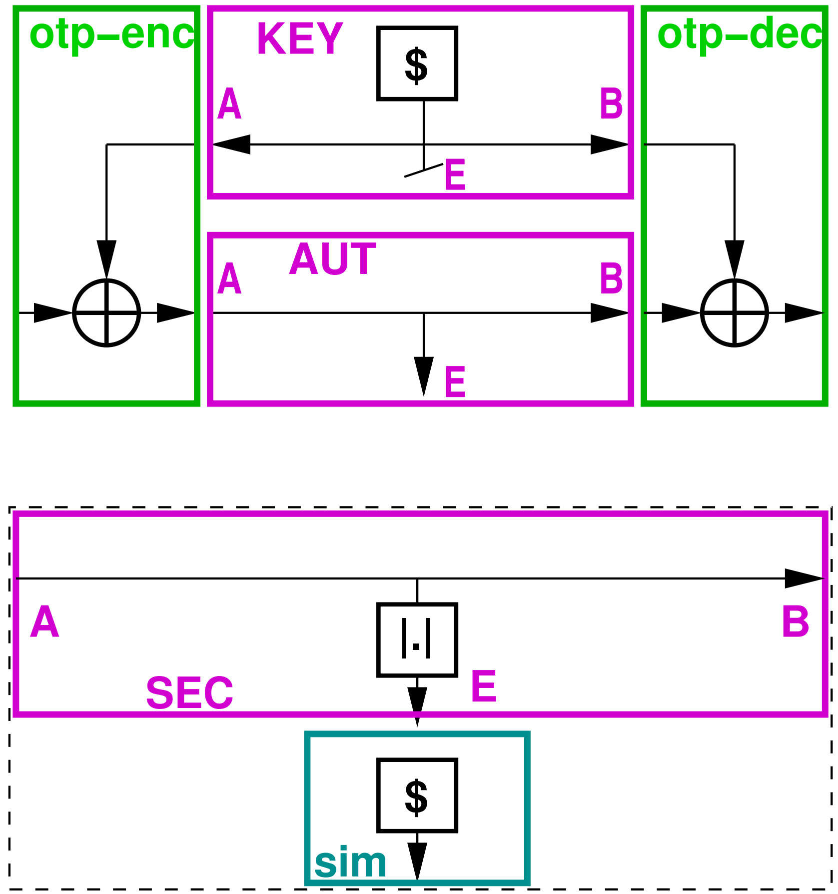
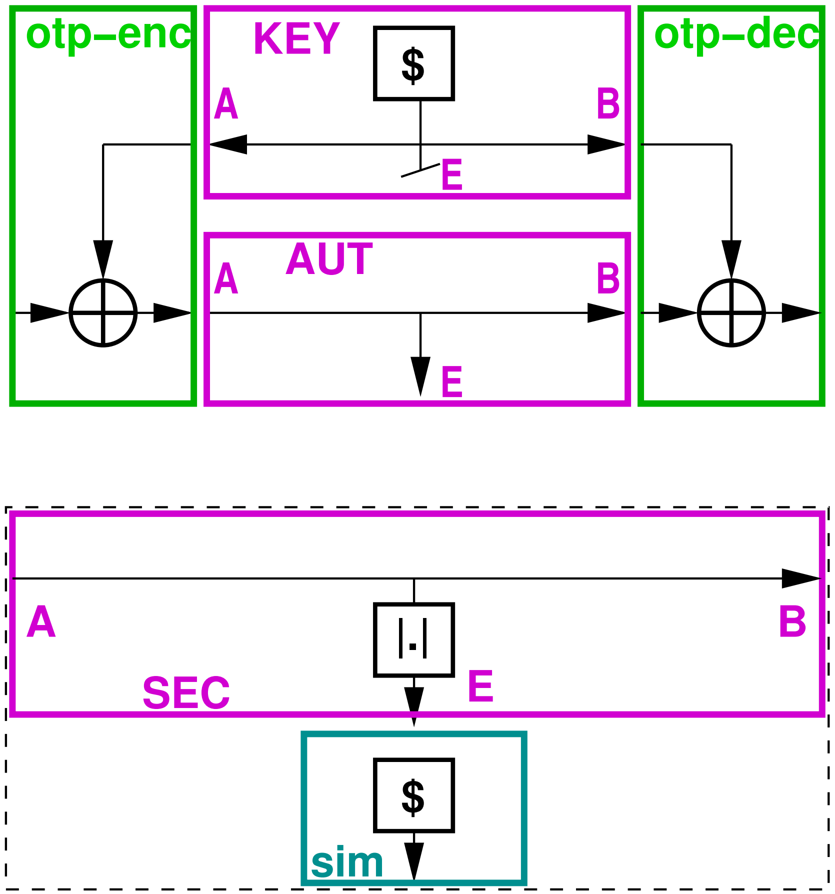


$$\textbf{otp-dec}^{B}\ \textbf{otp-enc}^{A}\ [\textbf{KEY}, \textbf{AUT}] \equiv \textbf{sim}^{E}\ \textbf{SEC}$$

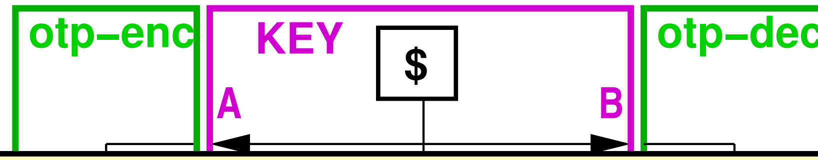
One-time pad in constructive cryptography



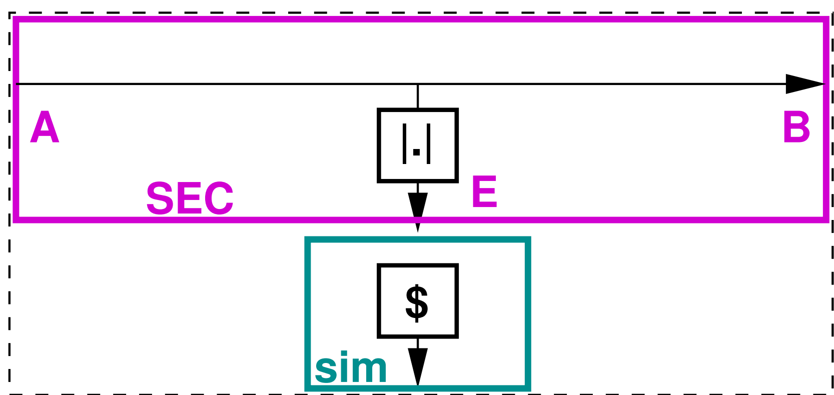
$$\text{otp-dec}^{B}\ \text{otp-enc}^{A}\ [\mathbf{KEY}, \mathbf{AUT}] \equiv \text{sim}^{E}\ \mathbf{SEC}$$

as a construction: $[\mathbf{KEY}, \mathbf{AUT}] \xrightarrow{\mathbf{OTP}} \mathbf{SEC}$

One-time pad in constructive cryptography



Draws on work by [Goldreich-Micali-Wigderson85], [Canetti01], [Pfitzmann-Waidner], [M.-Schmid96], ...



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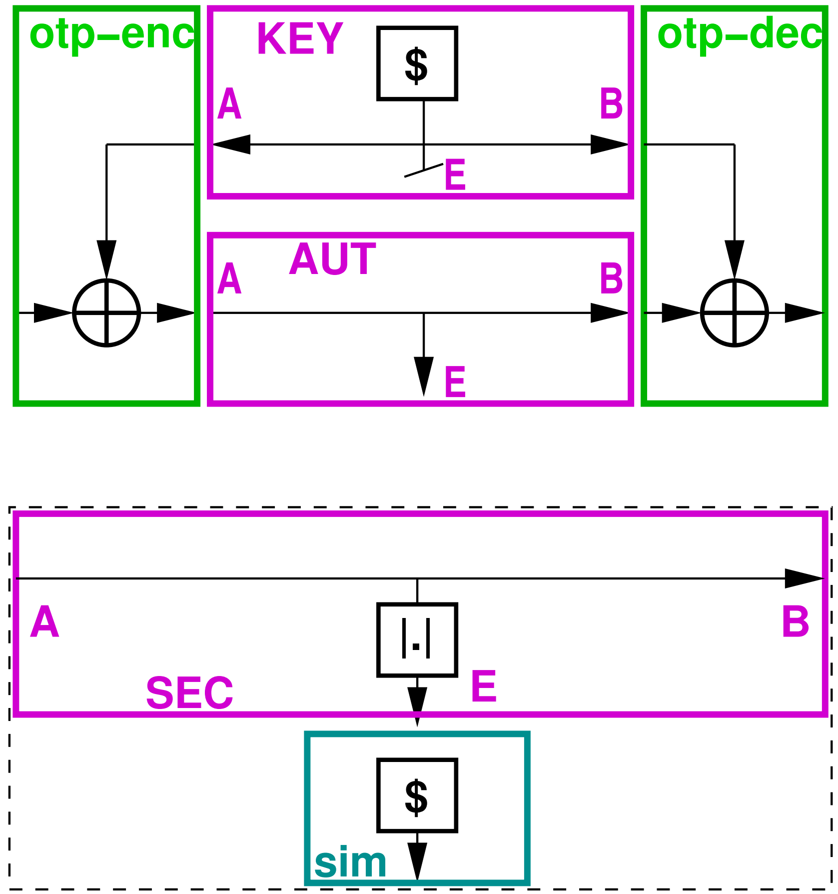

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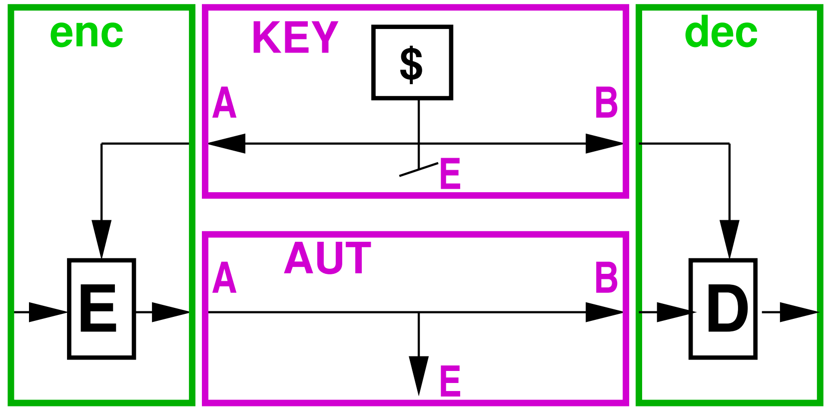
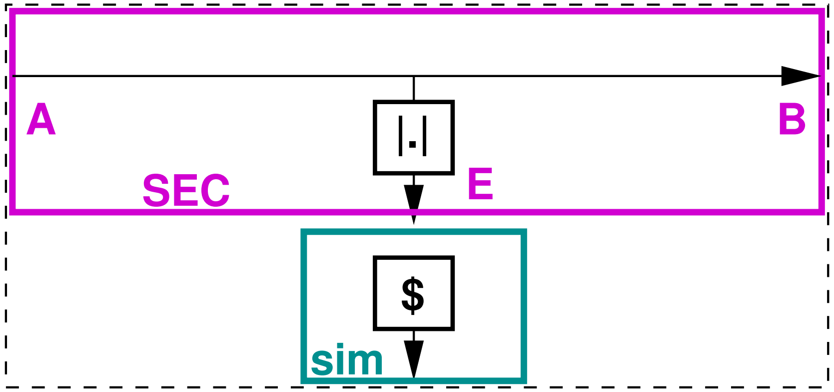
Encryption in constructive cryptography

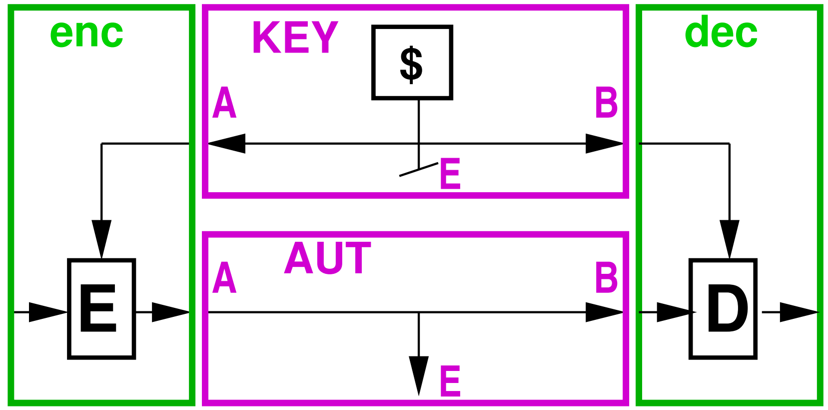


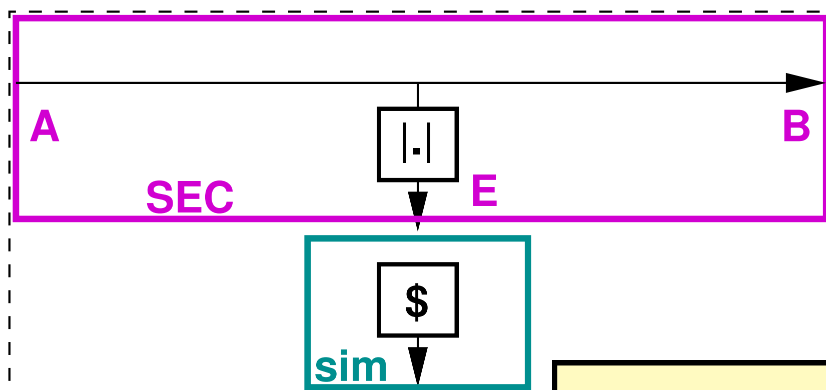


$$\mathbf{dec}^{B}\,\mathbf{enc}^{A}\,[\mathbf{KEY}, \mathbf{AUT}] \approx \mathbf{sim}^{E}\,\mathbf{SEC}$$

as a construction: $[\mathbf{KEY}, \mathbf{AUT}] \xrightarrow{\mathbf{SYM}} \mathbf{SEC}$

Encryption in constructive cryptography





metric?

$$\mathbf{dec}^{\mathsf{B}}\,\mathbf{enc}^{\mathsf{A}}\,[\mathbf{KEY}, \mathbf{AUT}] \approx \mathbf{sim}^{\mathsf{E}}\,\mathbf{SEC}$$

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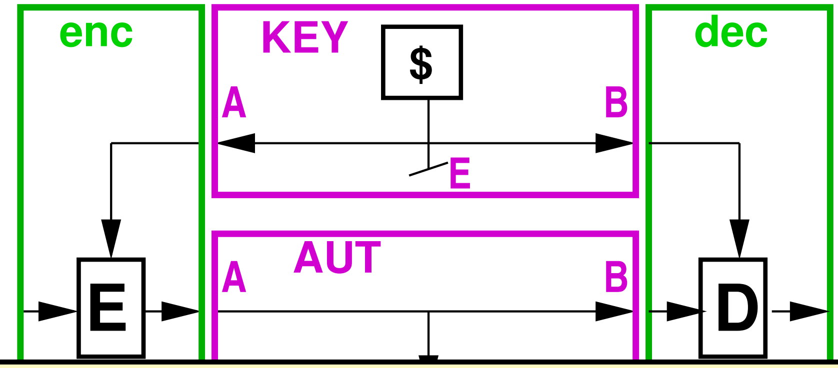
Encryption in constructive cryptography



$$\mathbf{R} \xrightarrow{(\pi_1,\pi_2)} \mathbf{S} \quad :\Leftrightarrow \quad \exists\sigma : \ \pi_1^{\mathsf{A}} \pi_2^{\mathsf{B}} \mathbf{R} \approx \sigma^{\mathsf{E}} \mathbf{S}$$

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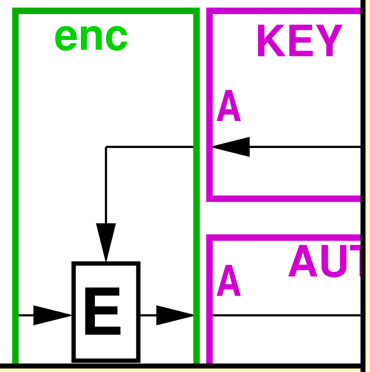
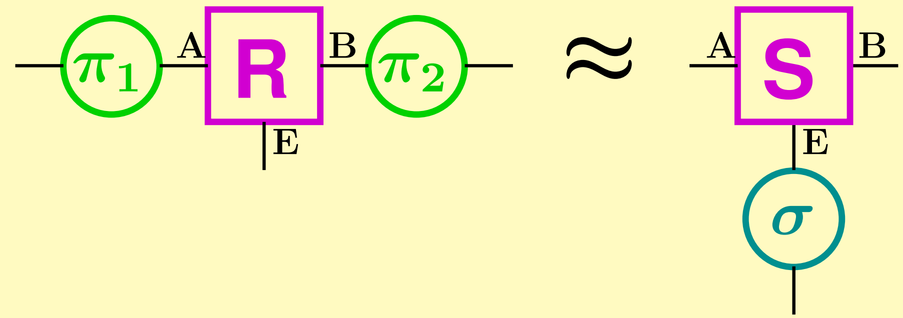
Encryption in constructive cryptography





$$\mathbf{R} \xrightarrow{(\pi_1,\pi_2)} \mathbf{S} \;:\Leftrightarrow\; \exists\sigma : \pi_1^{A}\,\pi_2^{B}\,\mathbf{R} \approx \sigma^{E}\,\mathbf{S}$$

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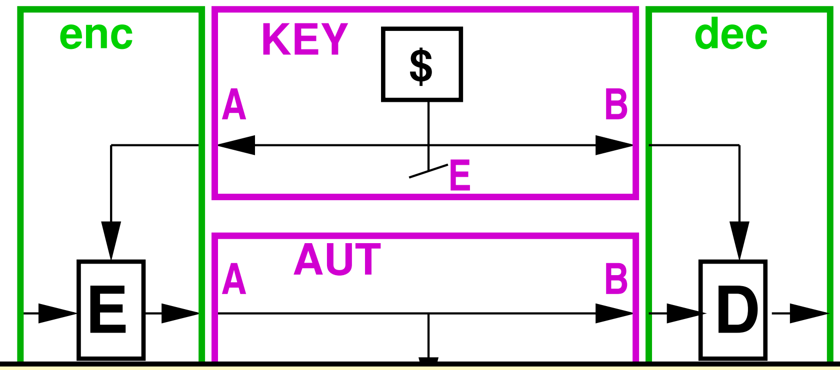

$$\mathbf{R} \xrightarrow{(\pi_1,\pi_2)} \mathbf{S} \;:\Leftrightarrow\; \exists\sigma : \pi_1^{\mathsf{A}}\,\pi_2^{\mathsf{B}}\,\mathbf{R} \approx \sigma^{\mathsf{E}}\,\mathbf{S}$$

and

$$\pi_1^{\mathsf{A}}\,\pi_2^{\mathsf{B}}\,\perp^{\mathsf{E}}\mathbf{R} \approx \perp^{\mathsf{E}}\mathbf{S}$$

$$\mathbf{dec}^{\mathsf{B}}\,\mathbf{enc}^{\mathsf{A}}\,[\mathbf{KEY},\mathbf{AUT}] \approx \mathbf{sim}^{\mathsf{E}}\,\mathbf{SEC}$$

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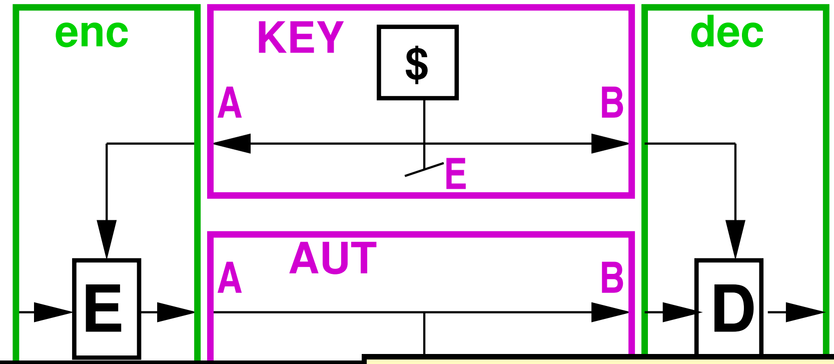
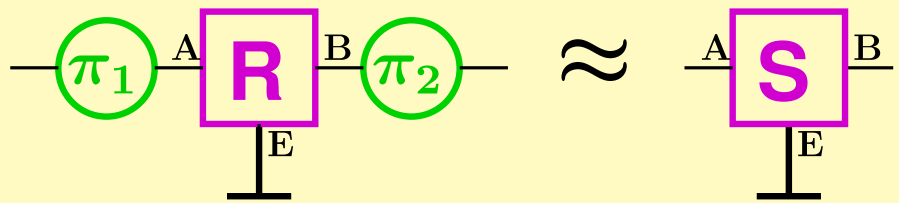
Encryption in constructive cryptography



$$R \xrightarrow{(\pi_1,\pi_2)} S \quad :\Leftrightarrow$$



$$\pi_1^{A}\ \pi_2^{B}\ \perp^{E}\ R \approx \perp^{E}\ S$$

$$\mathbf{dec}^{B}\ \mathbf{enc}^{A}\ [\mathbf{KEY}, \mathbf{AUT}] \approx \mathbf{sim}^{E}\ \mathbf{SEC}$$

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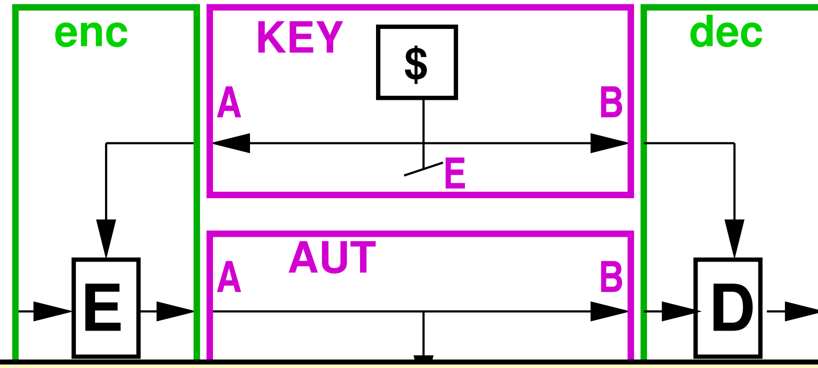

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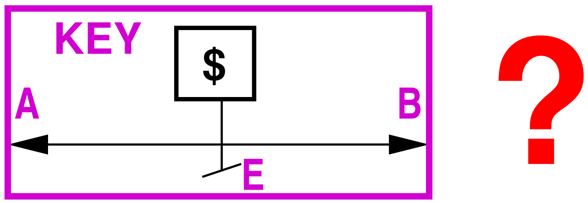
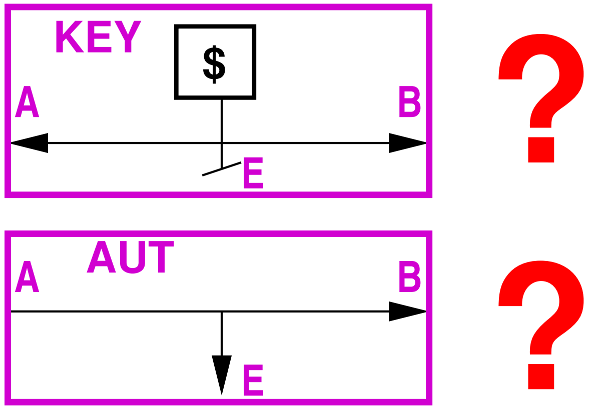
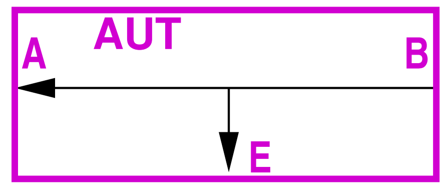
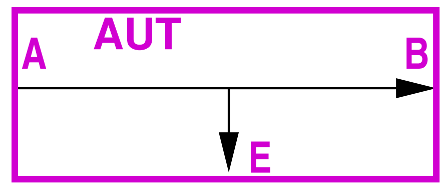


Encryption in constructive cryptography



Key agreement in CC





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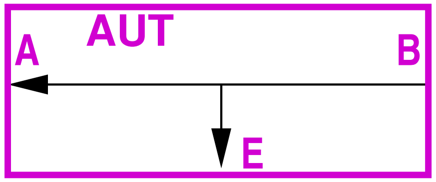

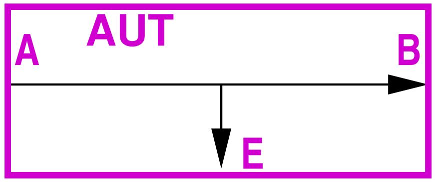

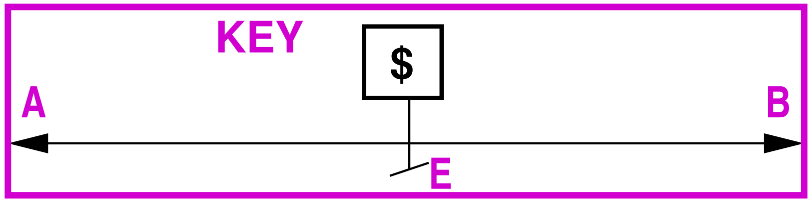

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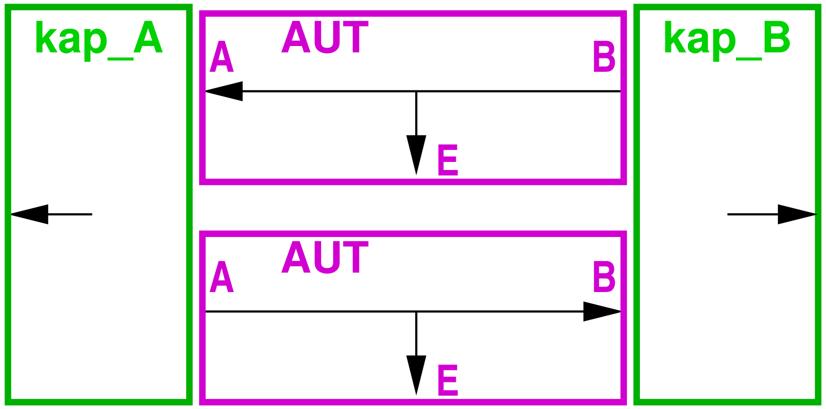

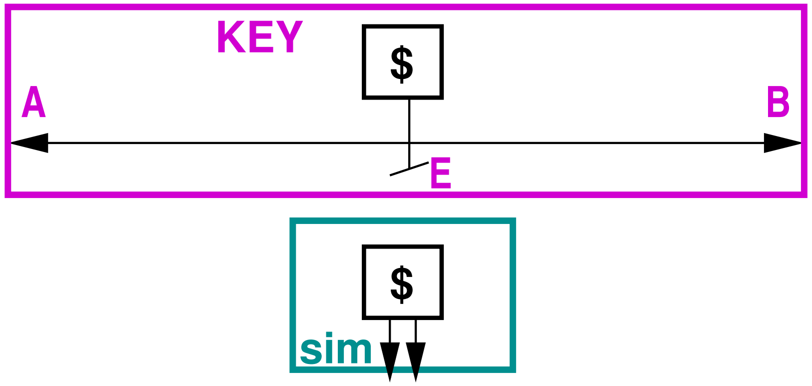

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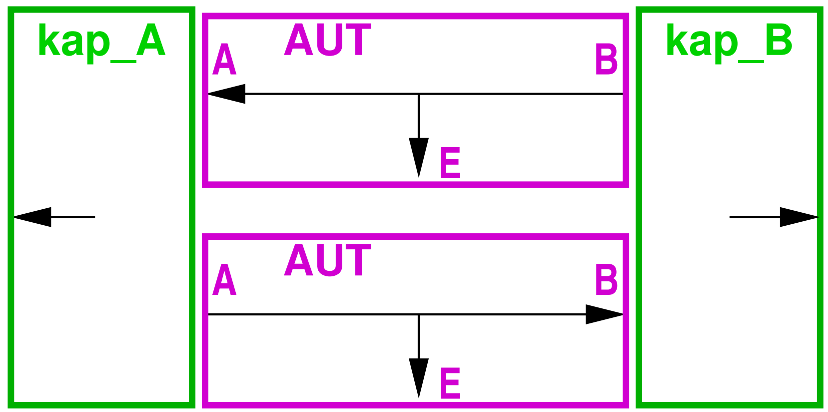
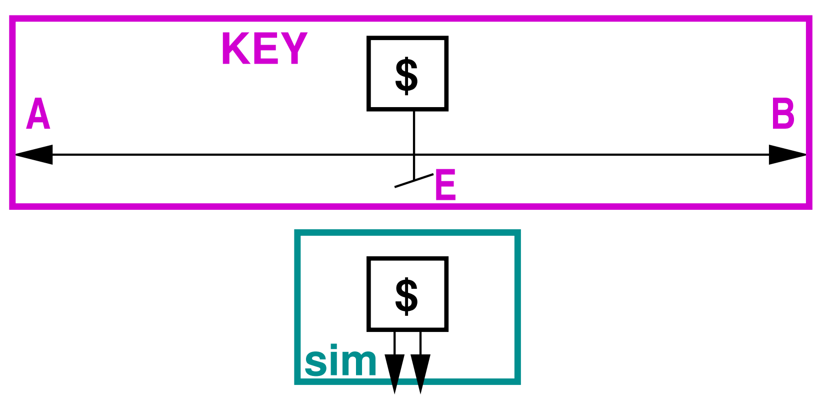
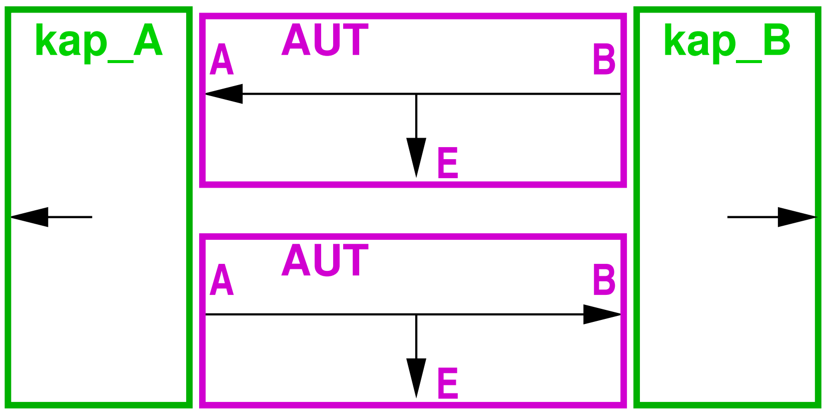
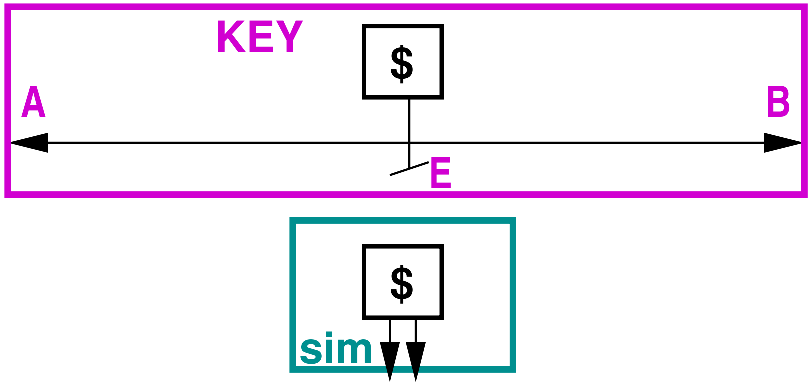




$$[\mathrm{AUT}, \mathrm{AUT'}] \xrightarrow{\mathrm{DifHel}} \mathrm{KEY}$$

Key agreement in CC (i.t. security)





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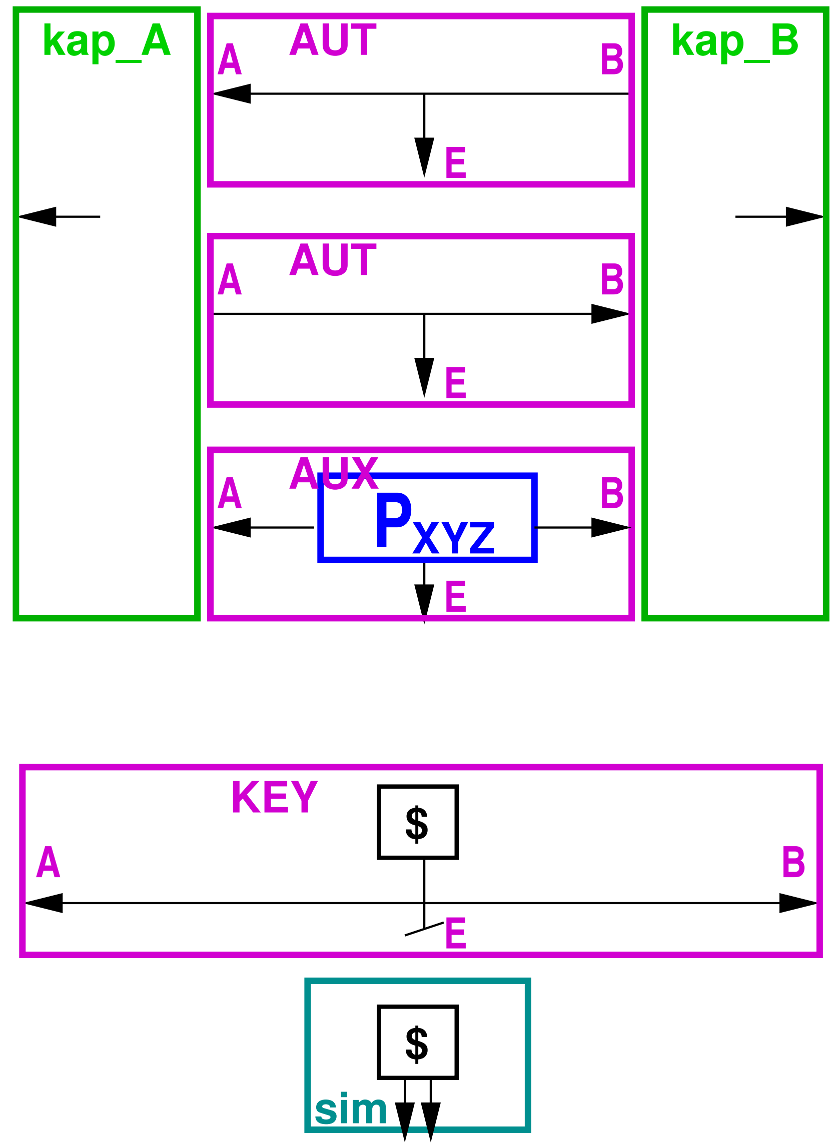

$$[\mathrm{AUT}, \mathrm{AUT'}, P_{XYZ}] \xrightarrow{\mathrm{KA\_PD}} \mathrm{KEY}$$

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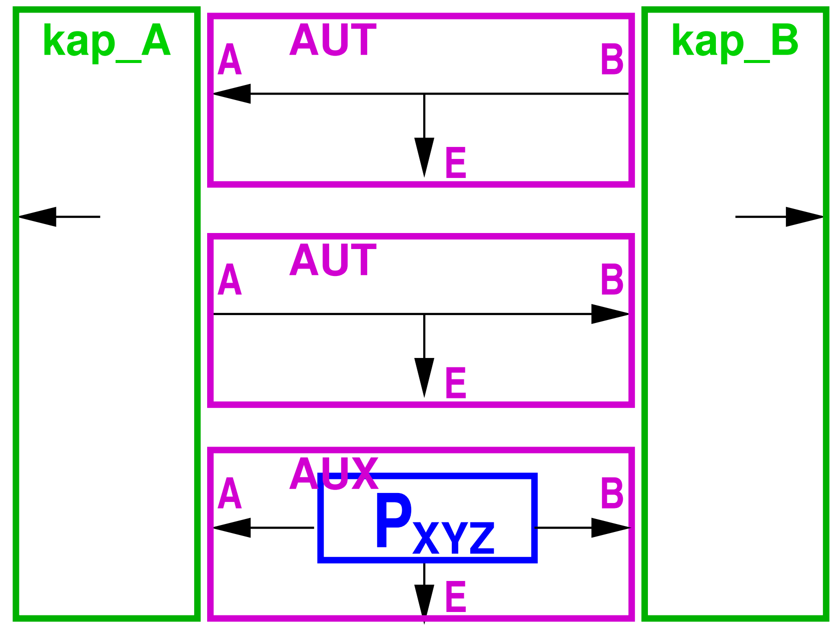
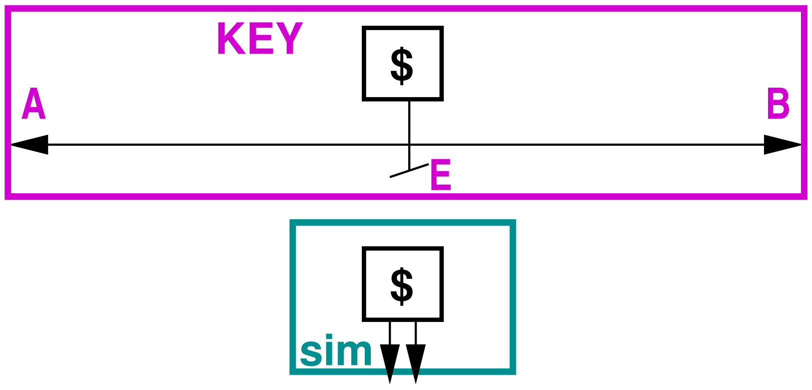
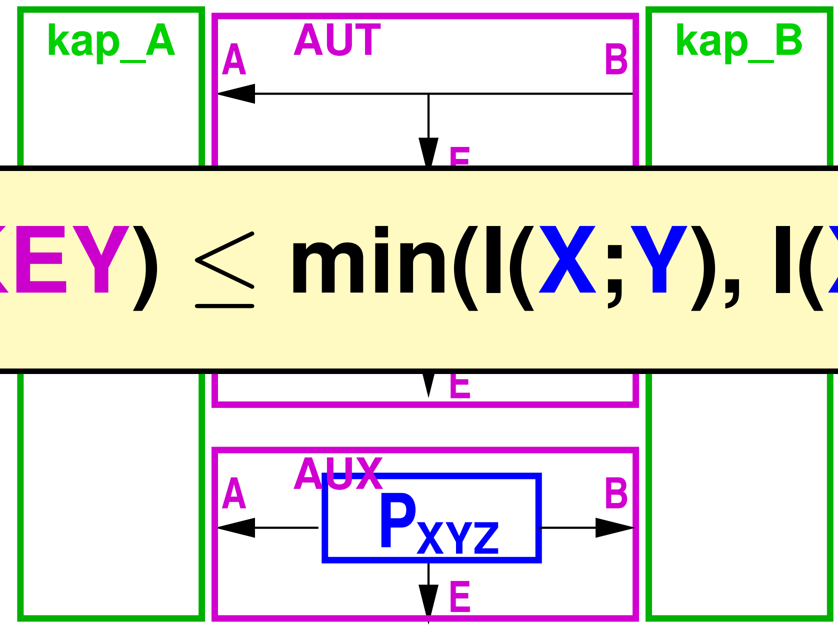


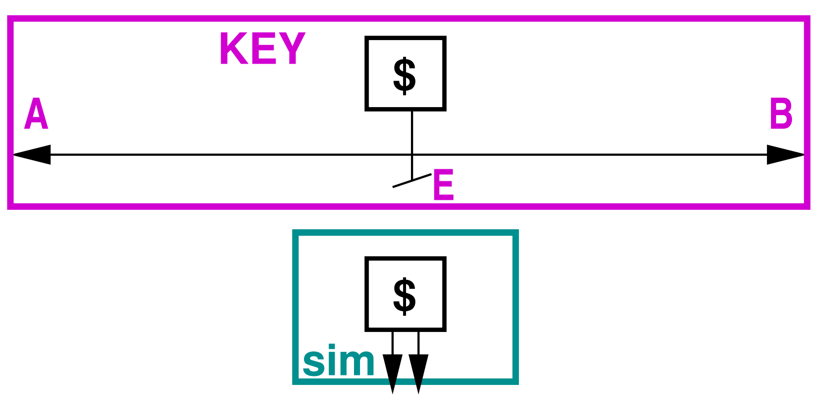


$$[\mathrm{AUT}, \mathrm{AUT'}, P_{XYZ}] \xrightarrow{\mathrm{KA\_PD},\ \epsilon} \mathrm{KEY}$$

Key agreement in CC (i.t. security)



Theorem: $H(\text{KEY}) \leq \min(I(X;Y), I(X;Y|Z))$ if $\epsilon = 0$.



$$[\text{AUT}, \text{AUT'}, P_{XYZ}] \xrightarrow{\text{KA\_PD}, \ \epsilon} \text{KEY}$$

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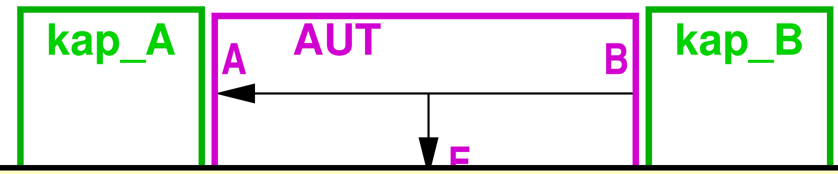

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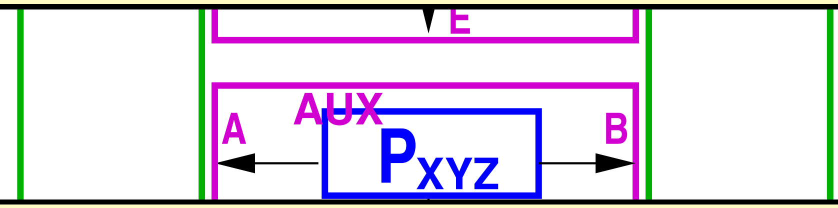
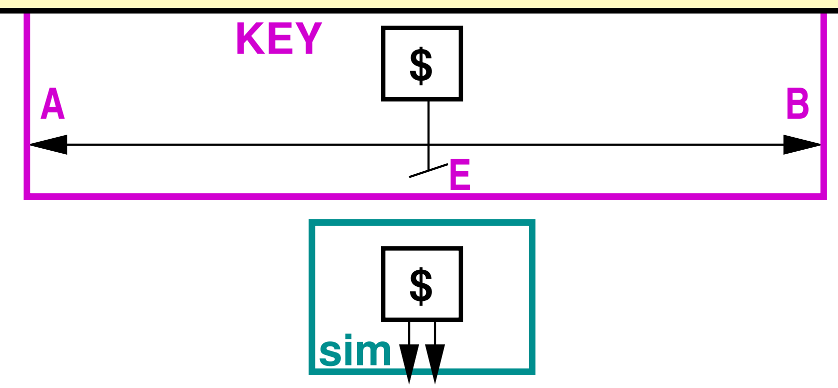


Theorem: $\epsilon \geq f(\, H(\text{KEY}) - I(X;Y|Z))$



$$[\text{AUT}, \text{AUT'}, P_{XYZ}] \xrightarrow{\text{KA\_PD}, \, \epsilon} \text{KEY}$$

Composition: an example

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$$\left.\begin{array}{l}[\text{AUT}, \text{AUT'}] \xrightarrow{\text{DifHel}} \text{KEY} \\ [\text{KEY}, \text{AUT''}] \xrightarrow{\text{SYM}} \text{SEC}\end{array}\right\} \Rightarrow$$

$$[[\text{AUT}, \text{AUT'}], \text{AUT''}] \xrightarrow{\text{DifHel} \circ \text{SYM}} \text{SEC}$$

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Attention: Quantum Key Distribution, though proven secure, did not compose before 2005 [KRBM07,Renner05]

Composition: an example

$[\text{AUT}, \text{AUT'}] \xrightarrow{\text{DifHel}} \text{KEY}$

$[\text{KEY}, \text{AUT''}] \xrightarrow{\text{SYM}} \text{SEC}$ $\Rightarrow$

$[\text{KEY}, \text{IC}, \text{QC}] \xrightarrow{\text{ITA}\circ\text{QKD}\circ\text{ITA}\circ\text{OTP}} \text{SEC}$??

$[\text{AUT}, \text{QC}] \xrightarrow{\text{QKD}\circ\text{OTP}} \text{SEC}$

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Proof of composition (for ABE-setting)

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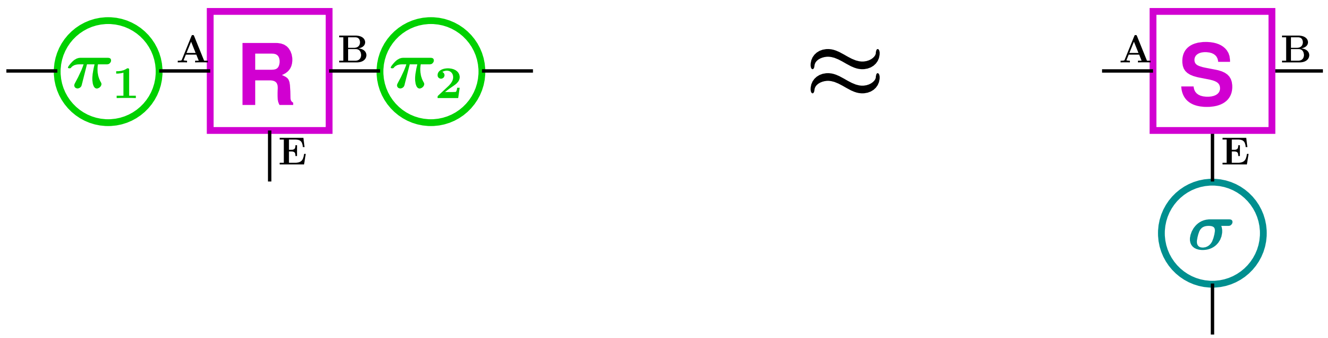
Definition: A construction is **composable** if

$$\mathbf{R} \xrightarrow{\alpha} \mathbf{S} \;\wedge\; \mathbf{S} \xrightarrow{\beta} \mathbf{T} \;\Rightarrow\; \mathbf{R} \xrightarrow{\alpha\circ\beta} \mathbf{T}$$



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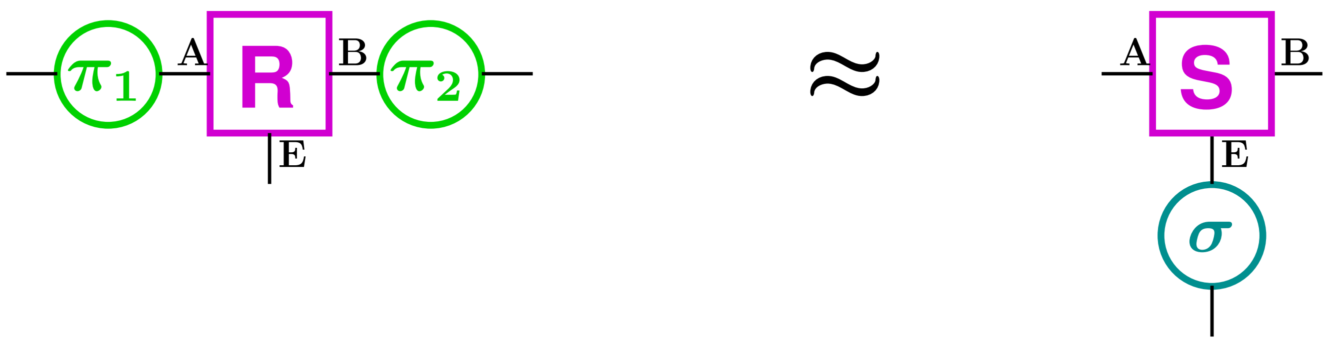

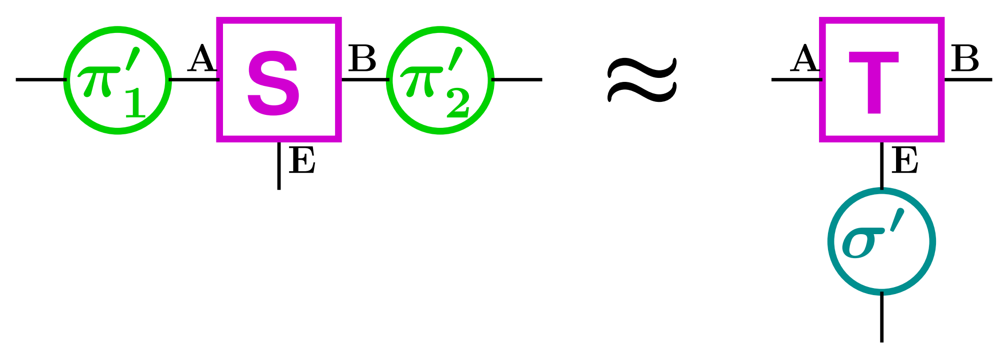

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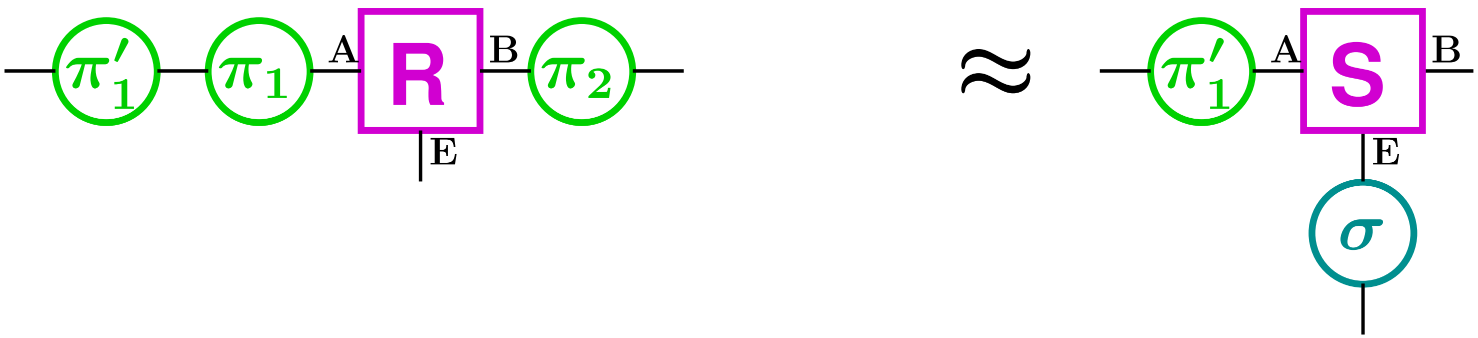

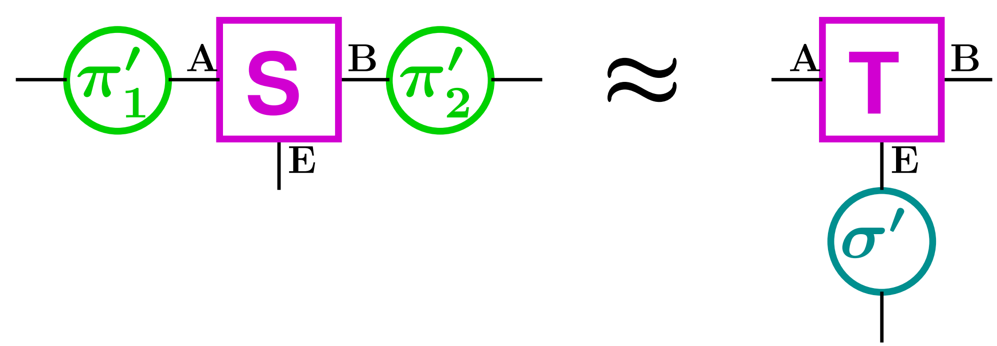

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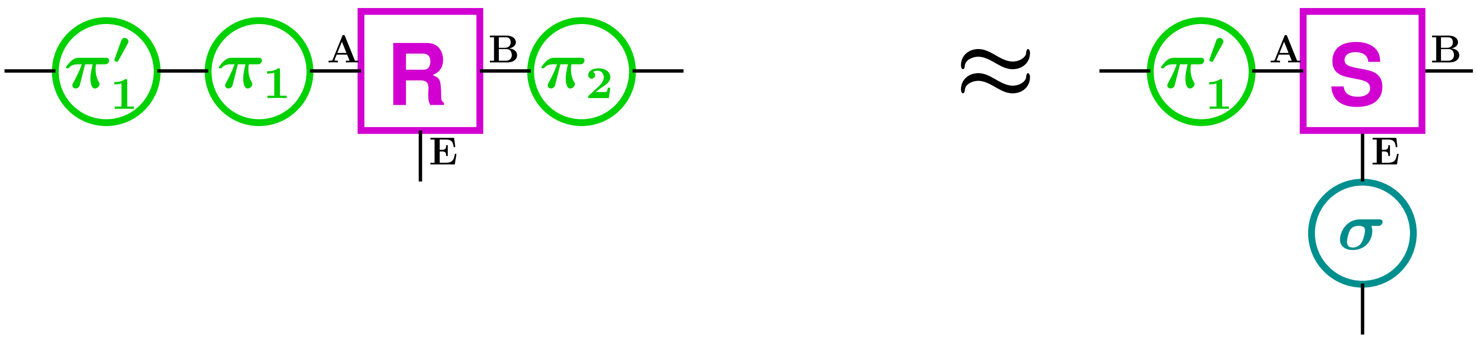
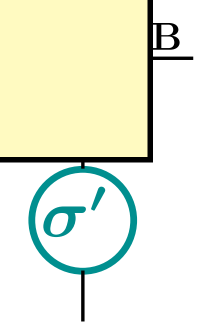


Pseudo-metric **d** on Φ is **non-expanding** if

$$\mathbf{d}(\gamma^i\mathbf{R},\gamma^i\mathbf{S}) \;\leq\; \mathbf{d}(\mathbf{R},\mathbf{S})$$



Proof of composition (for ABE-setting)

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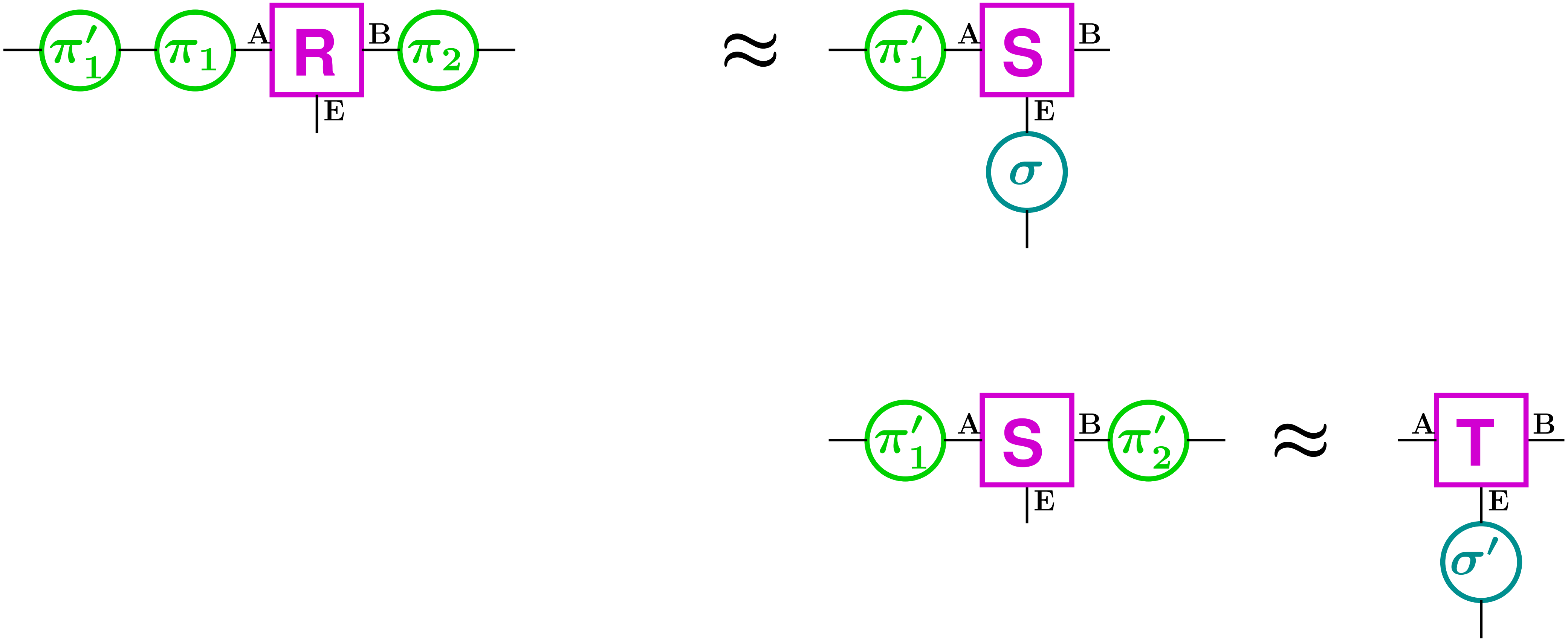

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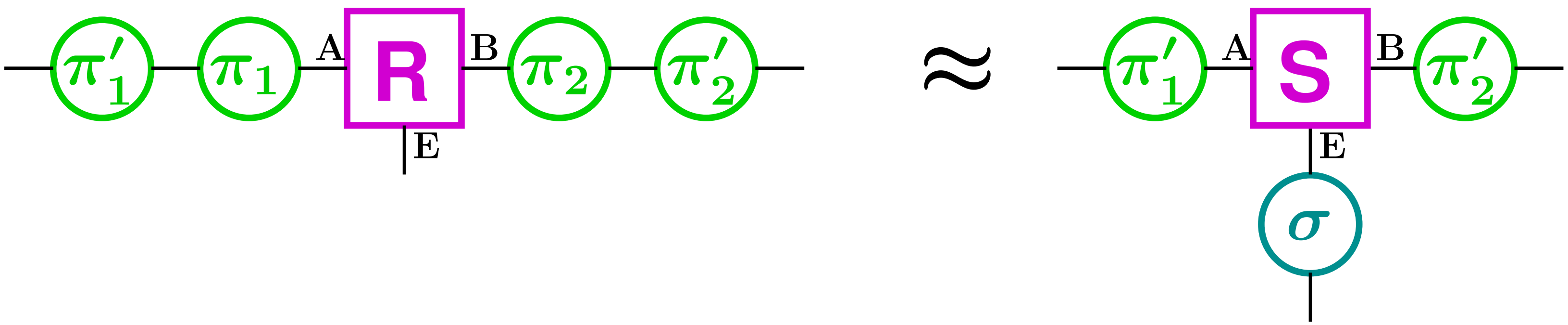

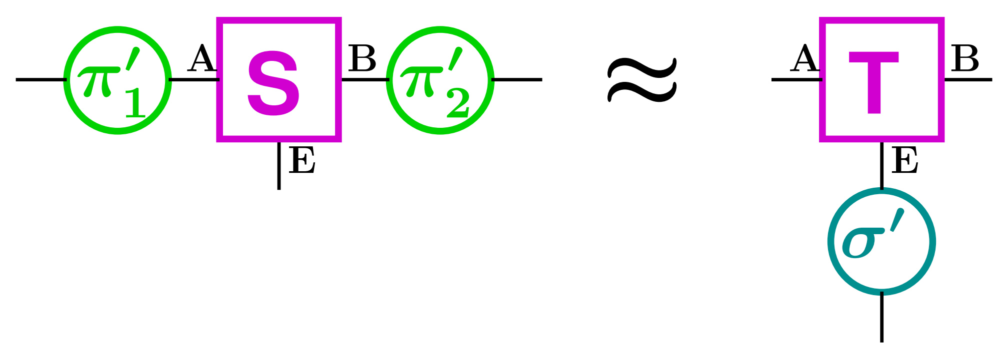

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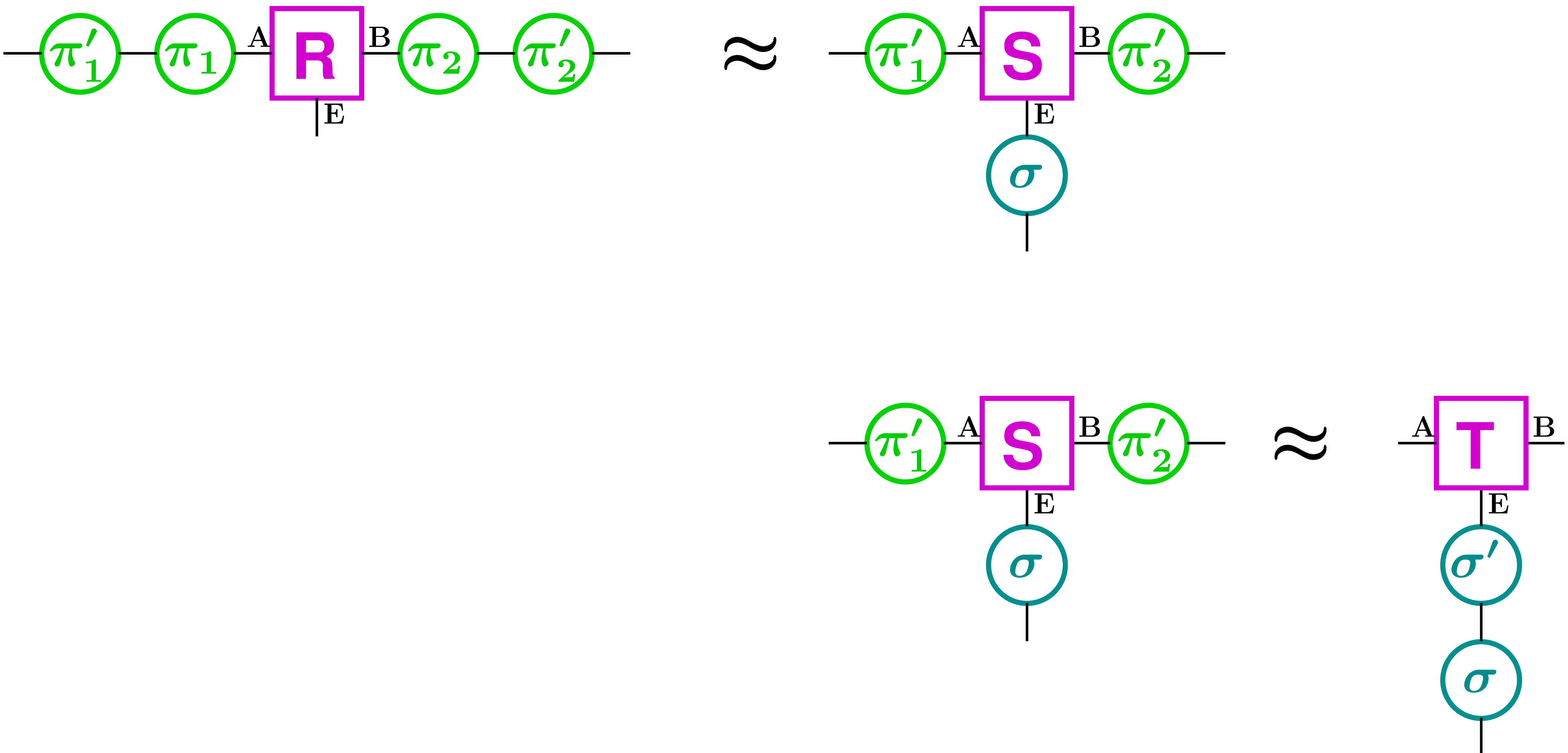

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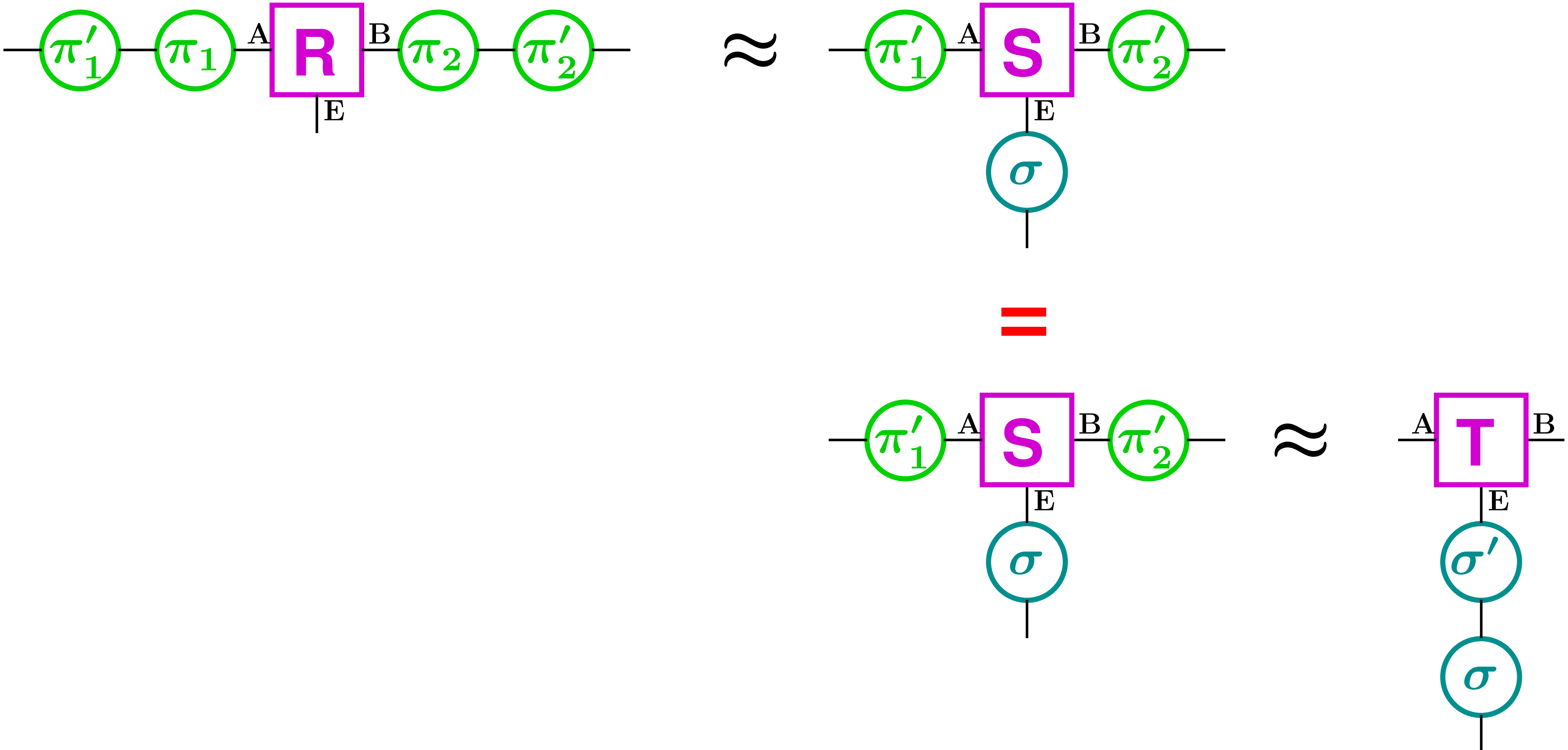

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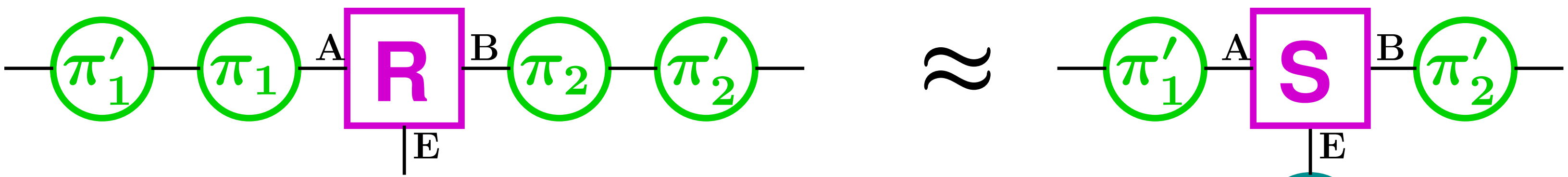
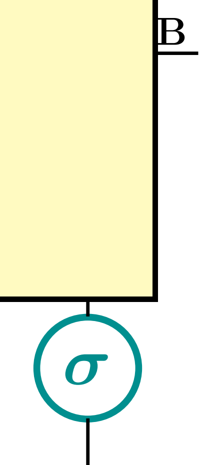


Generalizations of the ABE-setting:

- n≠3 parties
- any party can be dishonest



Thank you!

U. Maurer, **Authentication Theory and Hypothesis Testing**, *IEEE Trans. on Information Theory, 2005,*

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