

QCRYPT 2013

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IQC, University of Waterloo
Canada



Saturation Attack on Continuous-Variable Quantum Key Distribution System



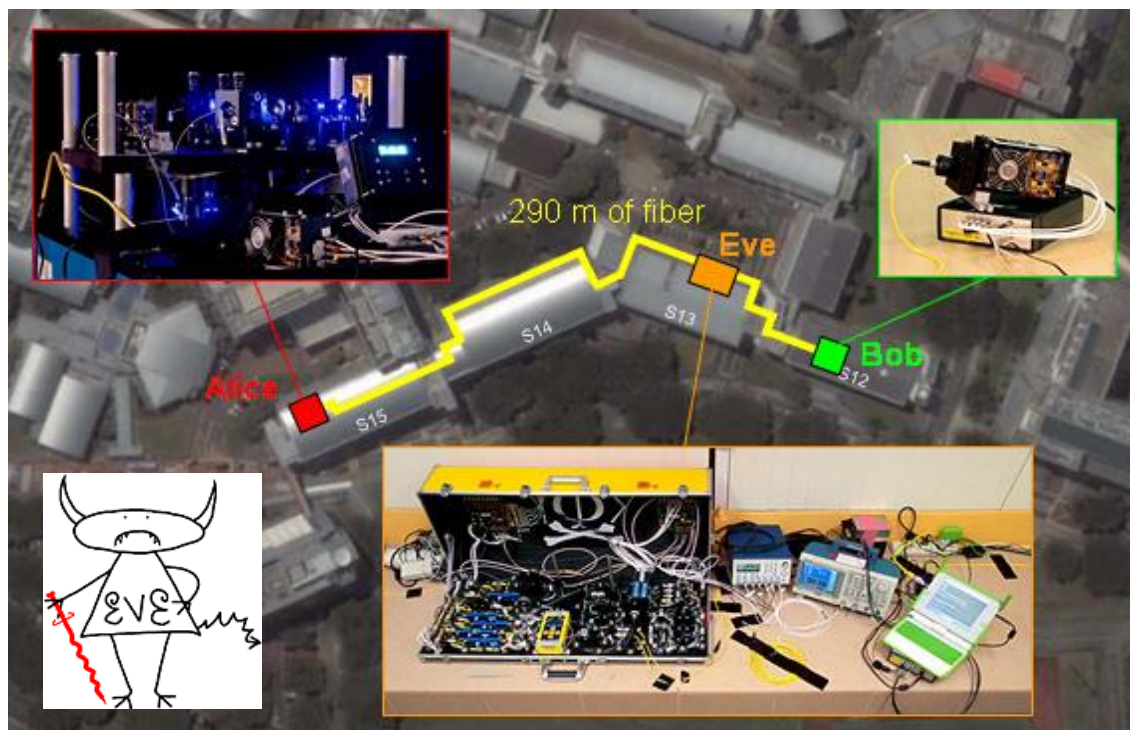
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Practical security of Quantum Key Distribution

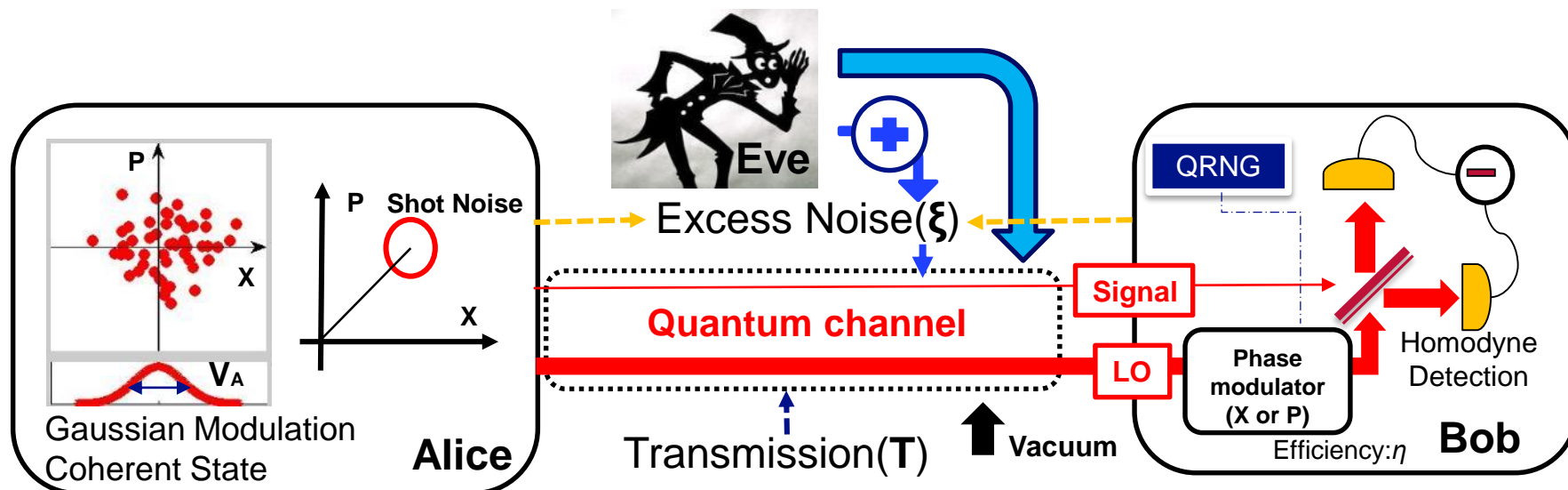
- Security of practical QKD depends on physical implementations
- Side channel attacks in DV QKD
- **Single photon detector is often a target**



Full-field implementation of a perfect eavesdropper on a quantum cryptography system

V. Makarov *et al.* , *Nature Comm.*2, 349 (2011)

Continuous-Variable Quantum Key Distribution



- Gaussian modulation coherent state (GMCS) protocol (F. Grosshans *et al.*, [Nature](#), 421:238–241, 2003)
- Quantum channel is totally characterized by T and ξ
- Side channel in CV QKD? Practical security?

Parameter Estimations in CV QKD

- **Gaussian linear model** $y = tx + z$ ($\sigma_z^2 = N_0 + \eta T \xi + v_{ele}$)

- Gaussian random variable: x : Alice, y : Bob, z : Noise, t : $\sqrt{\eta T}$

- $\langle x^2 \rangle = V_A$

- $\langle xy \rangle = \sqrt{\eta T} V_A$

- $\langle y^2 \rangle = \eta T V_A + N_0 + \eta T \xi + v_{ele}$

Calibration of shot noise (N_0) on Bob side (Alice and Bob close the quantum channel)

- $\langle y_0^2 \rangle = N_0 + v_{ele}$

Transmission

$$T = \frac{\langle xy \rangle^2}{\eta \langle x^2 \rangle^2}$$

Excess noise in shot noise unit (SNU)

$$\xi_{SNU} = \frac{\xi}{N_0} = \frac{\langle y^2 \rangle}{\eta T} - \frac{V_A}{N_0} - \frac{1}{\eta T} - \frac{v_{ele}}{\eta T N_0}$$

- **Secret key rate based on collective attack**

$$\Delta I = \beta I_{AB} - \chi_{BE}$$

Side channel attack in practical CV QKD system

■ Manipulation of Local oscillator

□ Equal-amplitude attack

- H.Häseler *et al.* [Phys. Rev. A 77, 032303 \(2008\)](#)

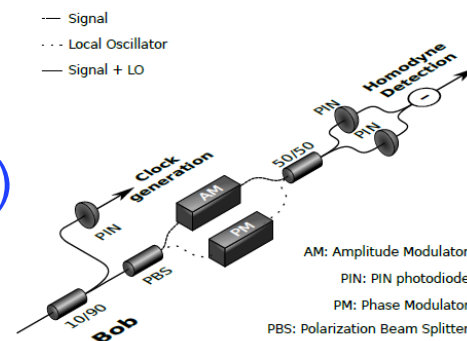
□ Calibration attack and preventing

- A. Ferenczi *et al.* [CLEOE-IQEC, Vol. 13 \(2007\)](#)
- P. Jouguet *et al.* [Phys. Rev. A 87, 062313 \(2013\)](#)
Influence on shot noise calibration.

□ LO fluctuation opens a loophole

- X-C. Ma *et al.* [arXiv:1303.6043 \(2013\)](#)
Inaccurate LO monitoring could lead to attack.

➤ Counter measure : Monitor LO Intensity & Real time shot noise calibration



Side channel attack in practical CV QKD system

■ Wavelength attack

- X-C. Ma *et al.* [Phys. Rev. A 87, 052309 \(2013\)](#)
- J-Z. Huang *et al.* [Phys. Rev. A 87, 062329 \(2013\)](#)
- Wavelength dependent beam splitter
- **Attack is possible even if LO is monitored.**



➤ Counter measure :

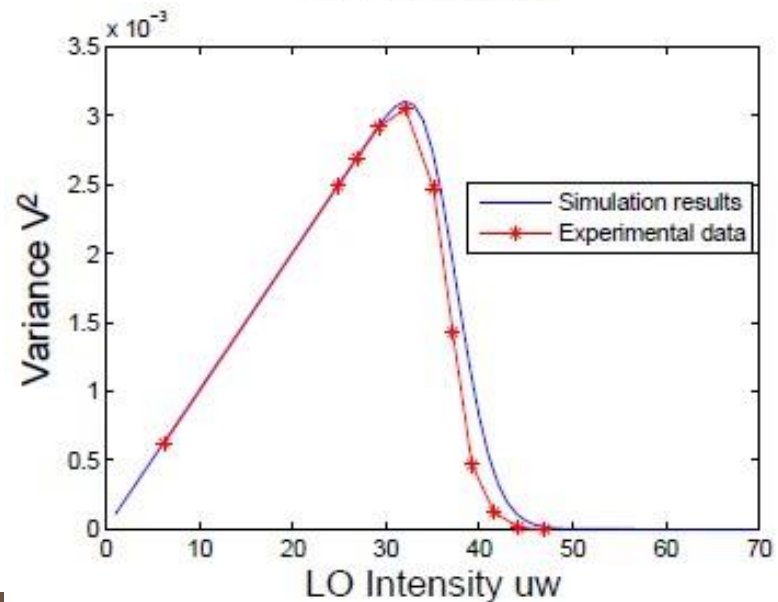
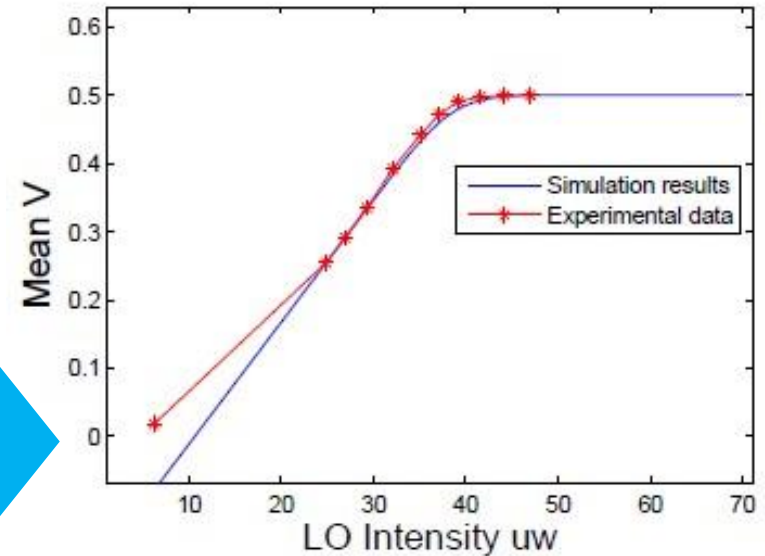
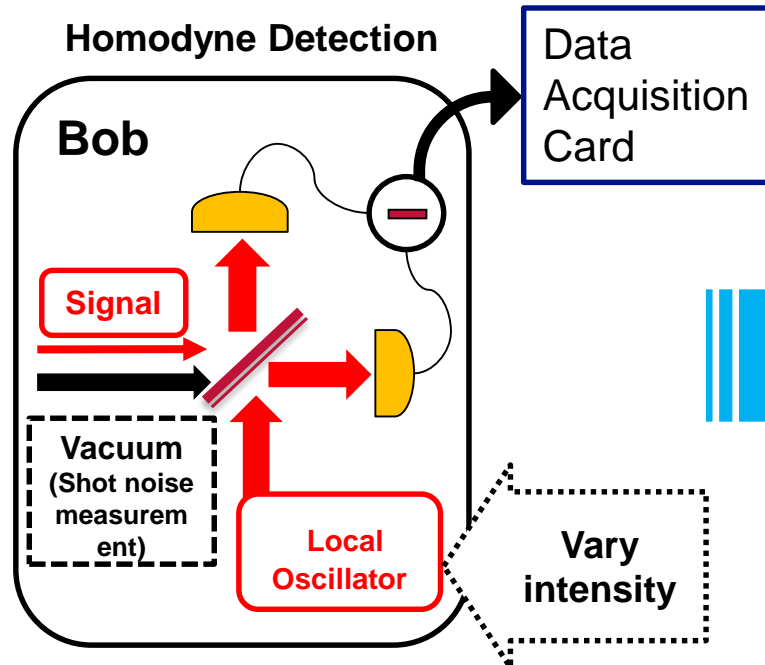
- Wavelength filter
- Wavelength independent beam splitter



✓ Combined with intercept resend attack

- ✓ Entanglement breaking: R. Namiki *et al.* [Phys. Rev. A 72, 024301 \(2005\)](#)
- ✓ Experimental demonstration of intercept resend attack on CV QKD: J. Lodewyck, *et al.* [Phys. Rev. Lett, 98, 030503 \(2007\)](#)

Experimental observation : Saturation of homodyne detection (Shot noise calibration)



- HD response saturates: **Violation** of the **assumption** in security proof of CV QKD
- Impact on the security? **New threat?**

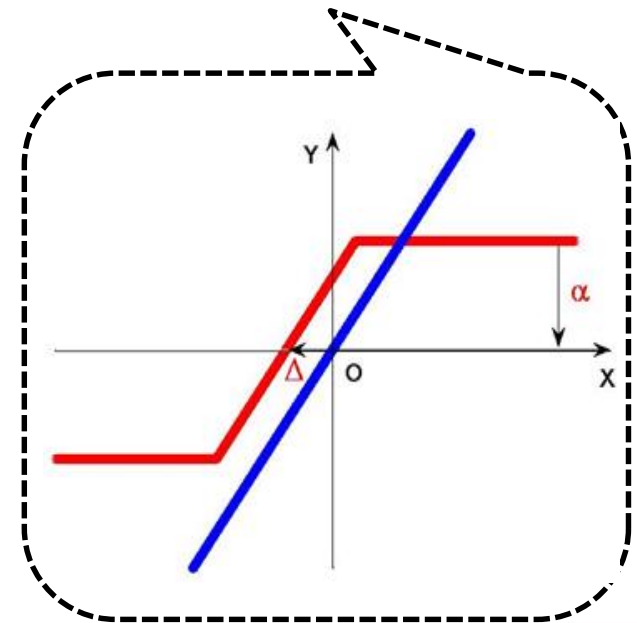
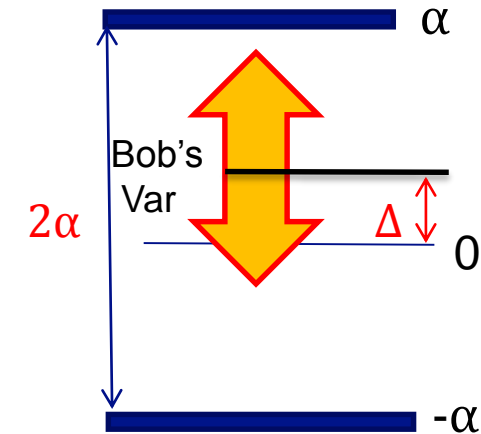
Saturation model

$$y_{sat} = \begin{cases} \alpha & y \geq \alpha \\ tx + z + \Delta & -\alpha < y < \alpha \\ -\alpha & y \leq -\alpha \end{cases}$$

- Detection range can not be infinity
- α is a **characteristic** of the detector
- Δ can be **manipulated** by Eve
- When α is large and Δ is small, saturation model returns to Gaussian linear model

$$y = tx + z$$

Homodyne detector's
Data acquisition card



What happens when there is saturation ?

■ **Saturation case:** $y_{sat} = \begin{cases} \alpha & y \geq \alpha \\ tx + z + \Delta & -\alpha < y < \alpha \\ -\alpha & y \leq -\alpha \end{cases}$

Analysis in the low saturation region ($\alpha^2 \gg V_B, \alpha^2 \gg N_0$):

■ $\langle x^2 \rangle = V_A$ unchanged

■ $\langle xy_{sat} \rangle = \sqrt{\eta T_{sat}} V_A \quad T_{sat} < T$

□ $\langle xy_{sat} \rangle \approx \frac{1}{2} \left[1 + \operatorname{erf} \left(\frac{\alpha - \Delta}{\sqrt{2\sigma_Z^2 + 2t^2\sigma_X^2}} \right) \right] \langle xy \rangle$

■ $V_{B,sat} = \eta T' V_A + N_0 + \eta T' \xi' + v_{ele} \quad V_{B,sat} < V_B$

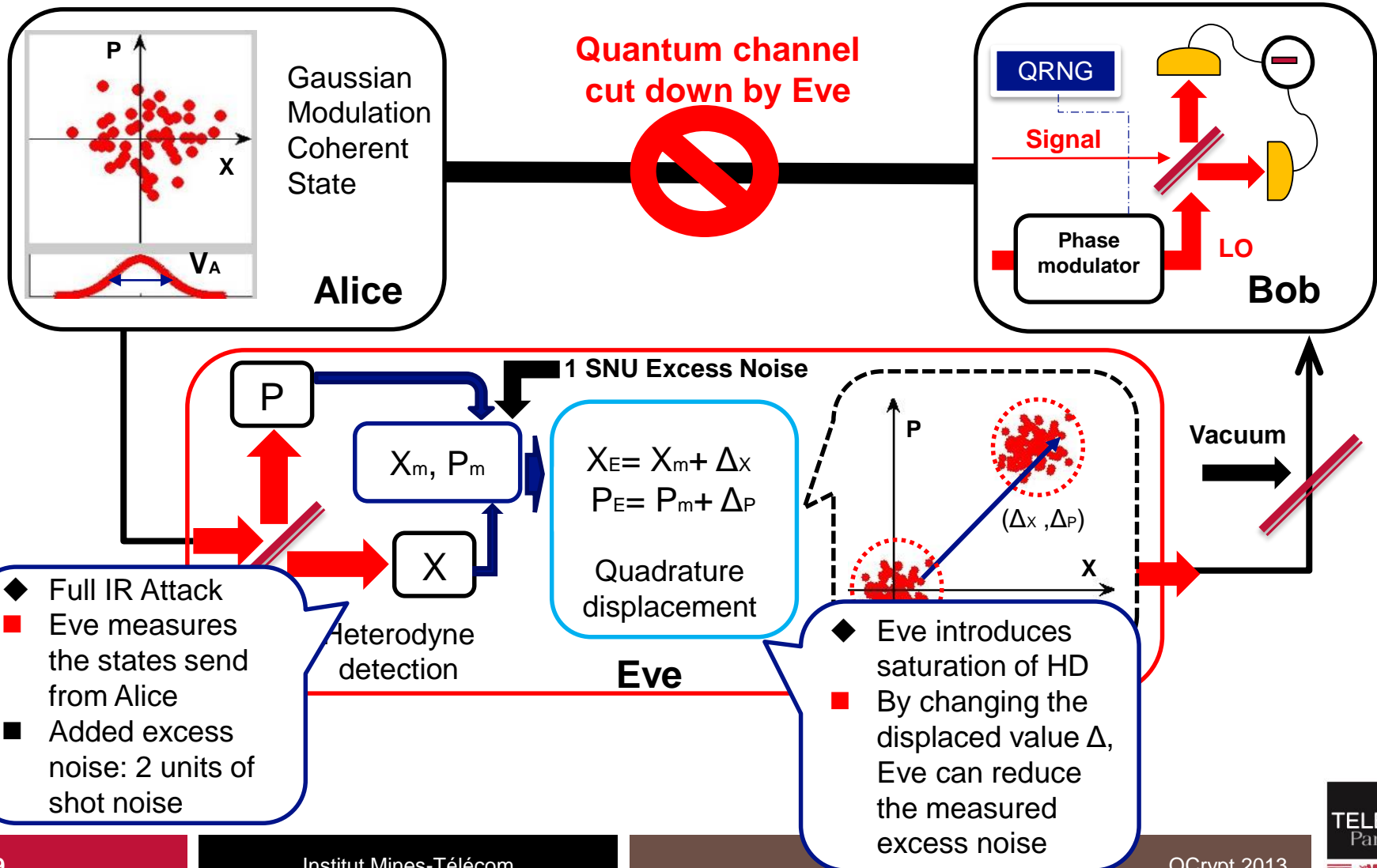
■ **Shot noise calibration:** $\langle y_0^2 \rangle = \langle z^2 \rangle \approx N_0 + v_{ele}$ unchanged

Excess noise in SNU will be changed and could be smaller

$$\xi'_{SNU} = \frac{\xi'}{N_0} = \frac{V_{B,sat}}{\eta T_{sat}} - \frac{V_A}{N_0} - \frac{1}{\eta T_{sat}} - \frac{v_{ele}}{\eta T_{sat} N_0}$$

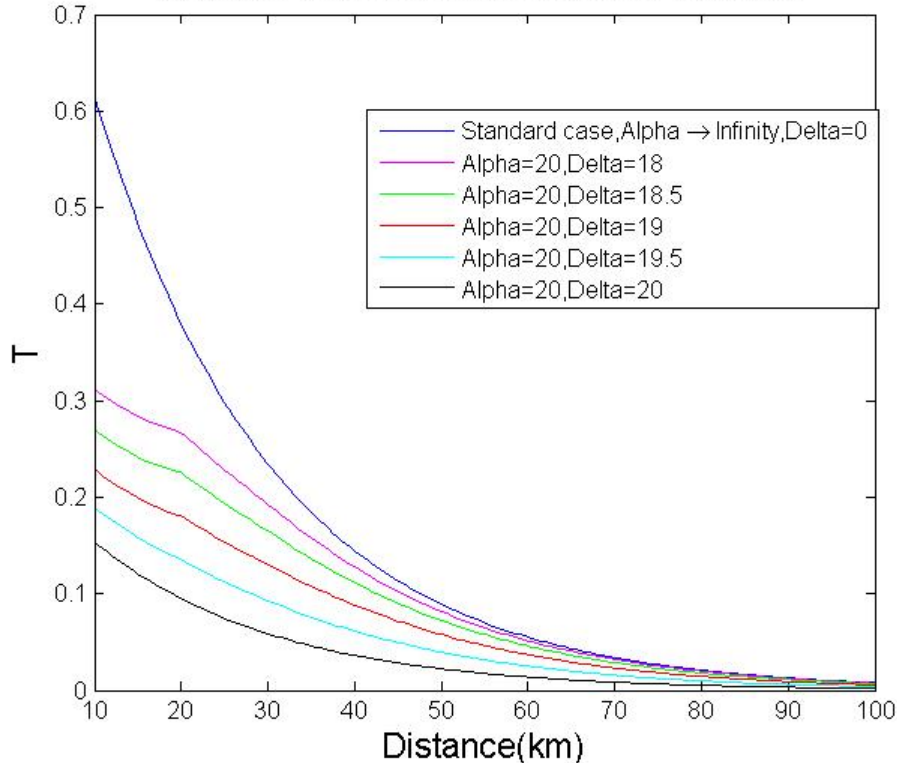
Saturation attack strategy

Full Intercept-resend attack+ Saturation of homodyne detection

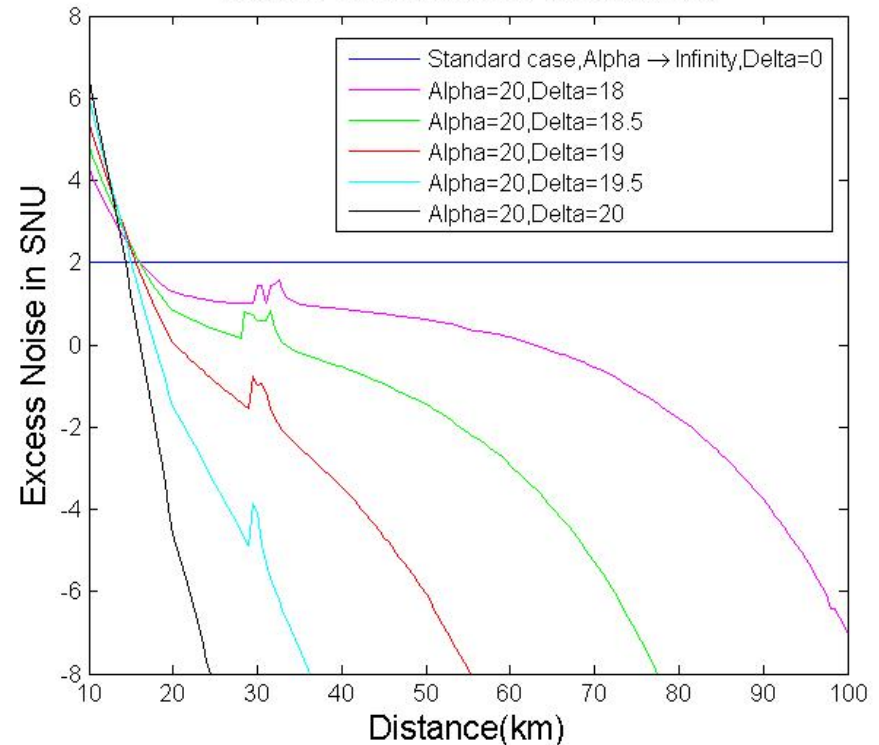


T & Excess noise estimations based on a Full IR Attack+ Saturation of homodyne detection (Δ)

Quantum channel transmission vs Distance



Excess Noise in SNU vs Distance

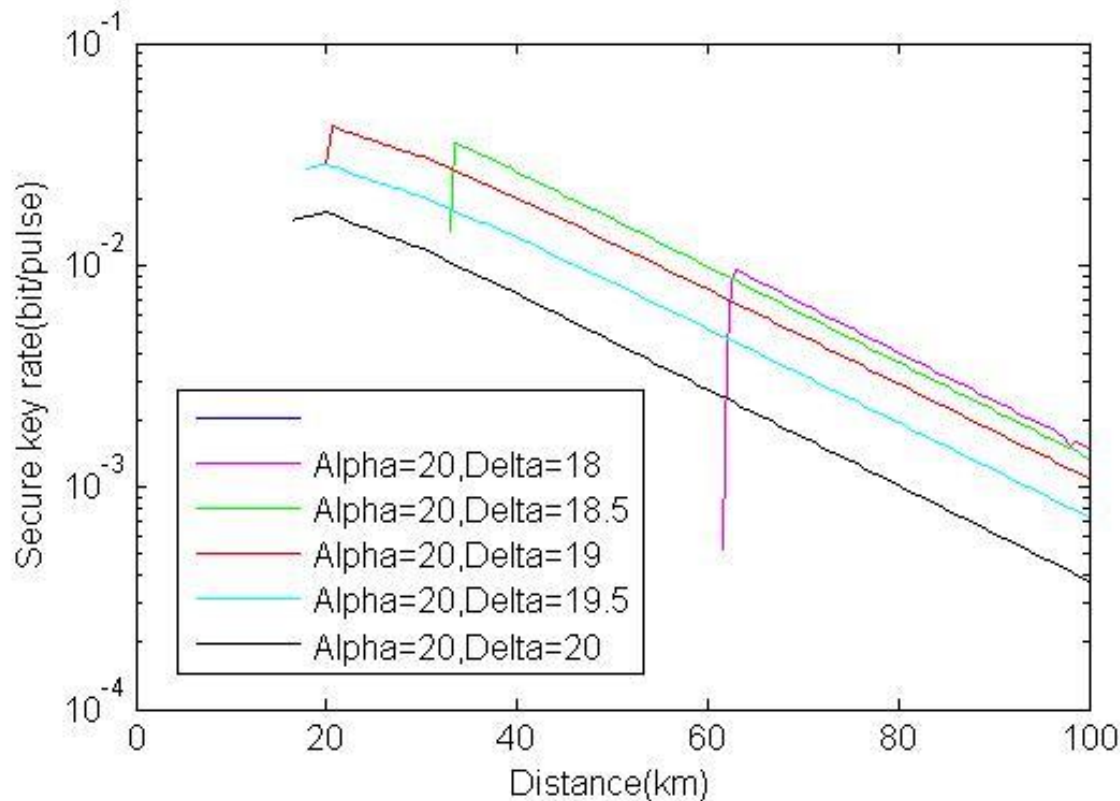


$V_A \in \{1, 100\}$, $\eta = 60\%$, $v_{ele} = 0.01$, $\xi_{sys} = 0.01$, $\xi_{Eve} = 2$, $\beta = 95\%$, $a = 0.21 \text{ dB/km}$

■ Excess noise can be reduced to an arbitrarily small value by changing Δ

Add noise if excess noise < 0

Secret key rate estimated by Alice and Bob based on a Saturation model+ Full IR Attack



Parameter setup

$$V_A \in \{1, 100\}$$

$$\eta = 60\%$$

$$v_{\text{ele}} = 0.01$$

$$\xi_{\text{sys}} = 0.01$$

$$\xi_{\text{Eve}} = 2$$

$$\beta = 95\%$$

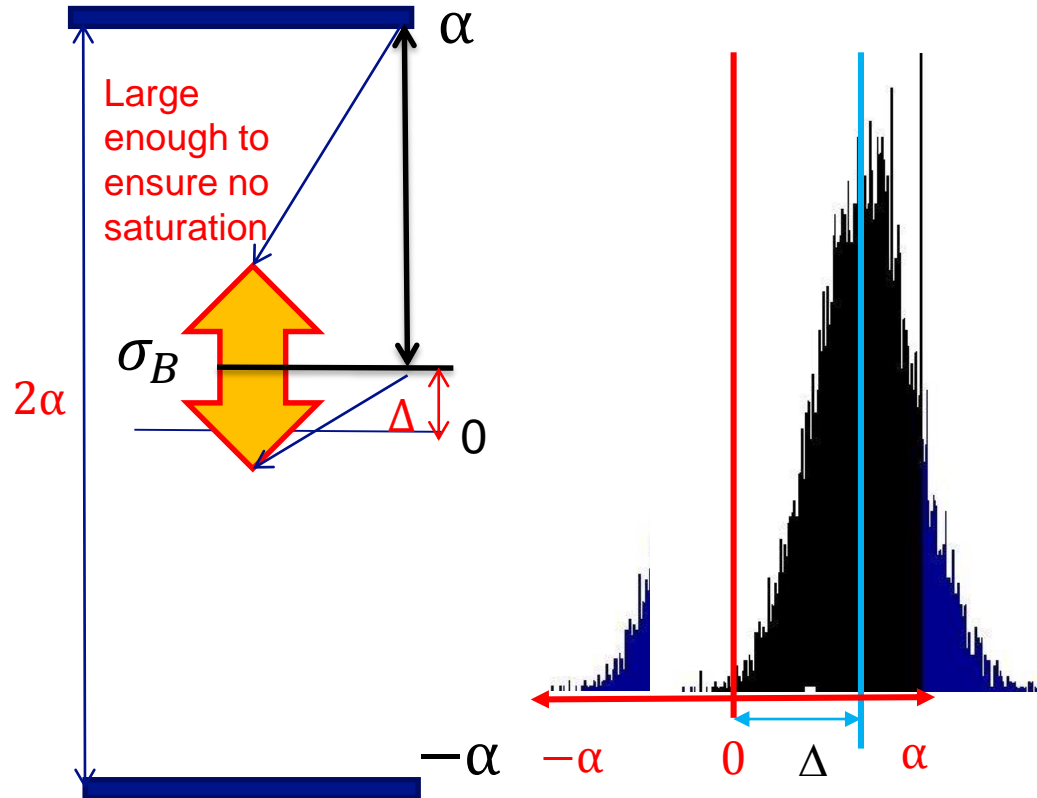
$$a = 0.21 \text{ dB/km}$$

Collective attack

$$\Delta I = \beta I_{AB} - \chi_{BE}$$


- ✓ Alice and Bob believe they still have some positive “secret key rate”. → **Effective attack, however T is reduced**
- Attack only possible above a distance which depends on Δ

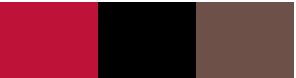
Counter measure



➤ Alice and Bob should add a test on the first moment (mean value) of X_B

Conclusions

- We have experimentally observed the saturation of homodyne detection
- Propose saturation attack which fully compromises the practical security of CV QKD system implemented GMCS protocol
- Saturation attack is achievable with current technology
- Assumptions in security proofs  Practical setup
- Propose suitable counter measures against saturation based attack

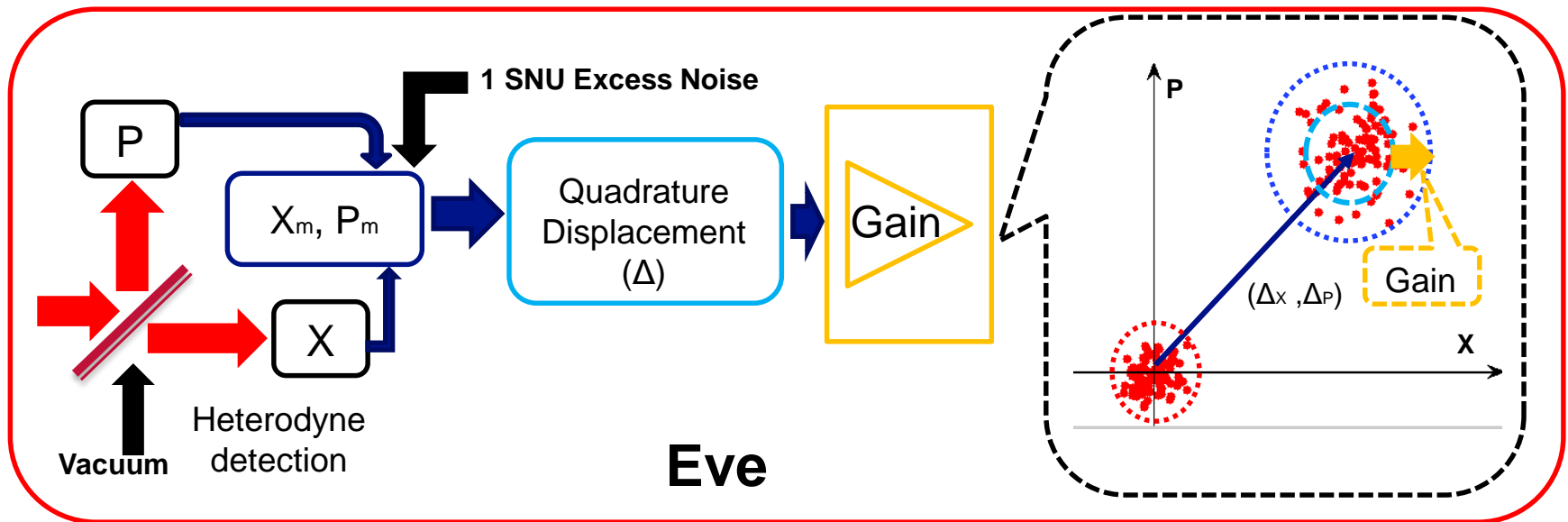


THANK YOU!

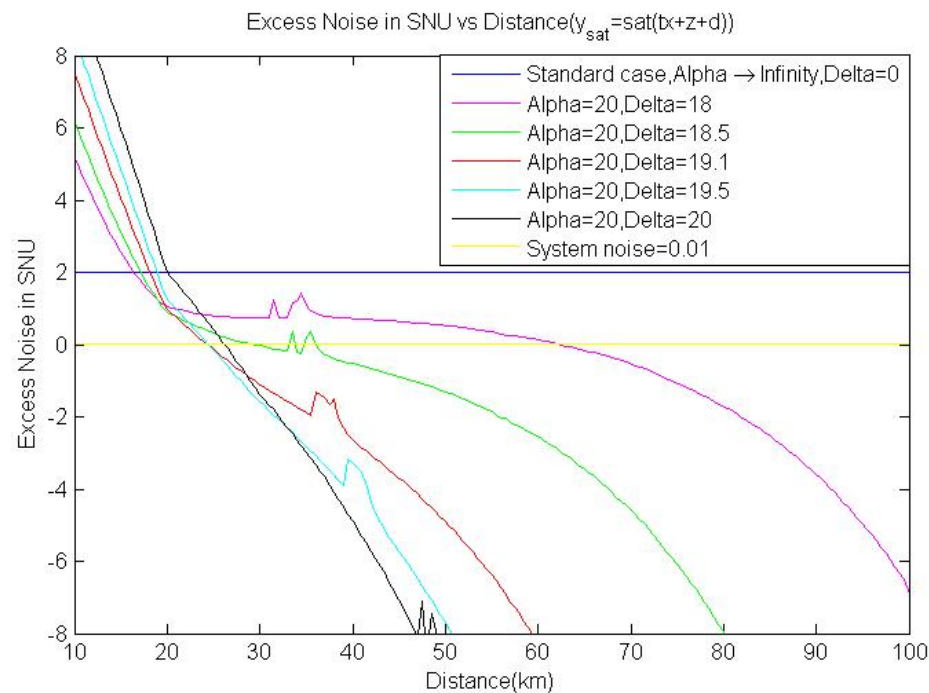
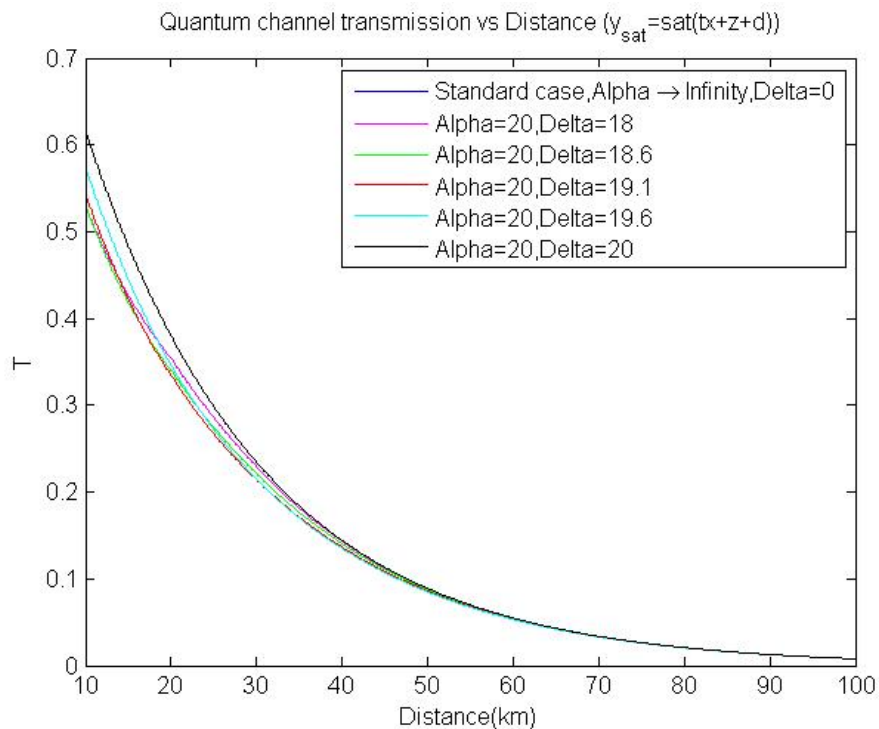
QUESTIONS?

Looking for zero-error attack: Improved strategy

- Eve amplifies the states that she sends to Bob
- Eve has chance to control both T and ξ



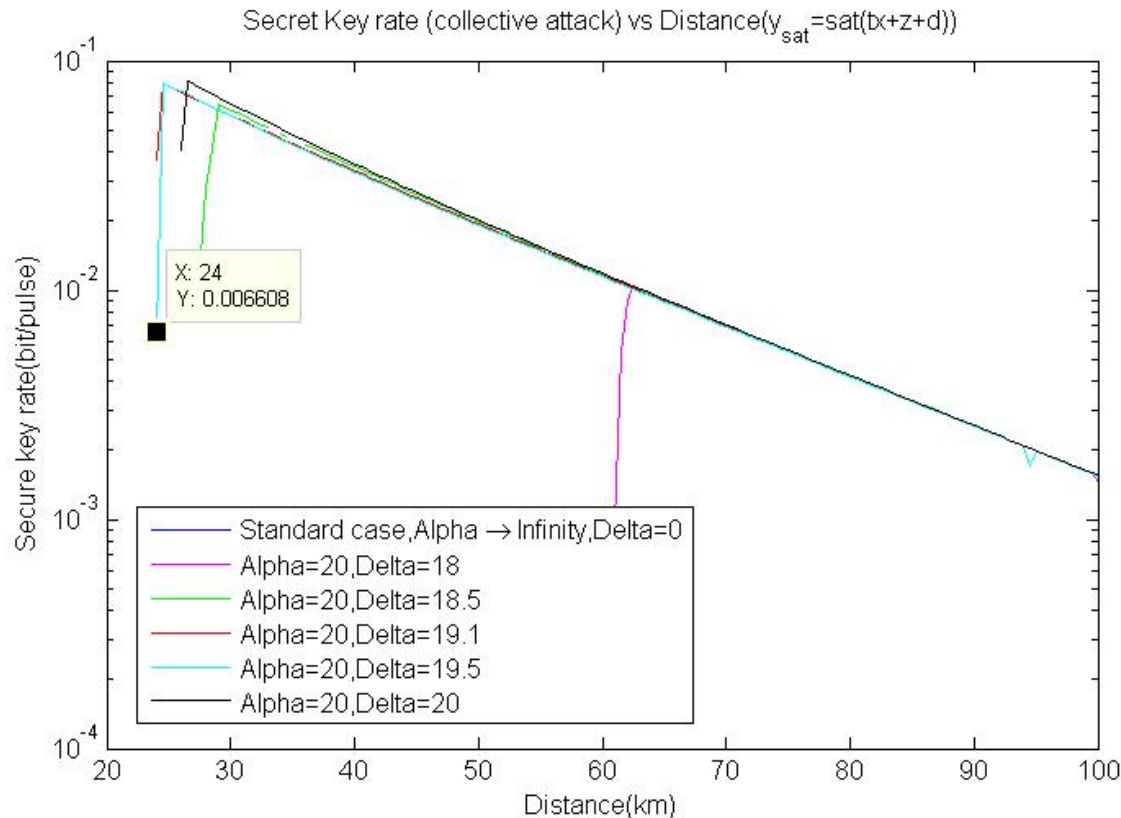
T & Excess noise estimations under improved strategy



$V_A \in \{1, 100\}$, $\eta_{\text{Bob}} = 60\%$, $v_{\text{ele}} = 0.01$, $\xi_{\text{sys}} = 0.01$, $\xi_{\text{Eve}} = 2$, $\beta = 95\%$, $a = 0.21 \text{ dB/km}$

✓ Quantum channel transmission is improved!

Secret key rate estimated by Alice and Bob under improved strategy



Parameter setup

$V_A \in \{1, 100\}$

$\eta = 60\%$

$V_{\text{ele}} = 0.01$

$\xi_{\text{sys}} = 0.01$

$\xi_{\text{Eve}} = 2$

$\beta = 95\%$

$a = 0.21 \text{ dB/km}$

Collective attack

$$\Delta I = \beta I_{AB} - \chi_{BE}$$

- Eve has achieved a “zero error” attack: T and ξ unchanged; Eve knows everything which Alice sends to Bob
- ✓ Key rate increases
- Attack distance from 24 km

Parameters taken for the simulations

- V_A , chosen according to Figure on the right (optimal choice of ECC, imposing a fixed SNR $\Rightarrow V_A$ (vs Distance)).
- Efficiency of Bob : $\eta=60\%$,
- Excess noise of electronics: $v_{elec}=0.01$
- Excess noise of system: $\xi_{sys}=0.01$
- Reconciliation efficiency: $\beta=95\%$
- Attenuation coefficient: $a=0.21\text{dB/km}$

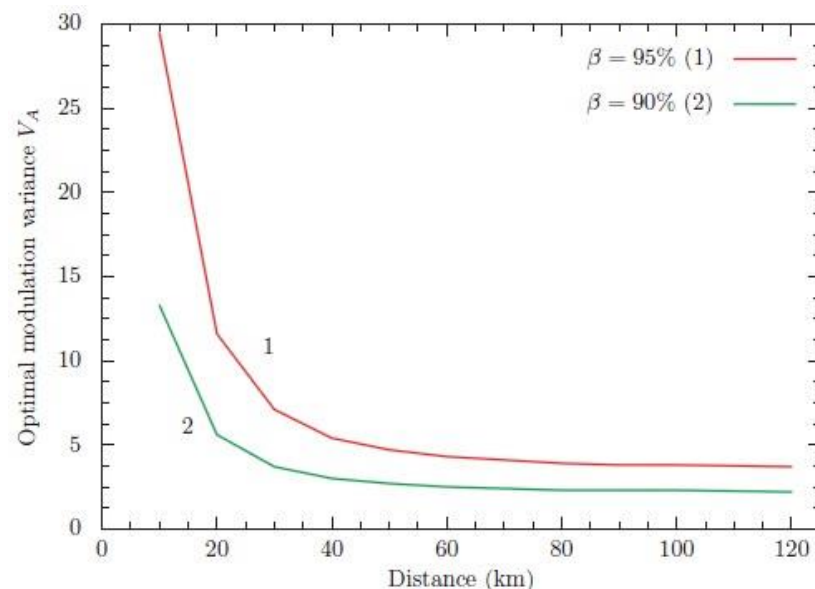


FIG. 2. (Color online) Optimal modulation variance with respect to the distance: $\eta = 0.6$, $V_{elec} = 0.01$, $\xi = 0.01$, $\alpha = 0.2\text{dB/km}$, and $\beta = 95\%, 90\%$ from top to bottom.

Long-distance continuous-variable quantum key distribution with a Gaussian modulation
[PRA 84, 062317, 2011](#)

Experimental demonstration of long-distance continuous-variable quantum key distribution
[Nature Photonics, 10, 1038, 2013](#)