Loopholes in Bell experiments

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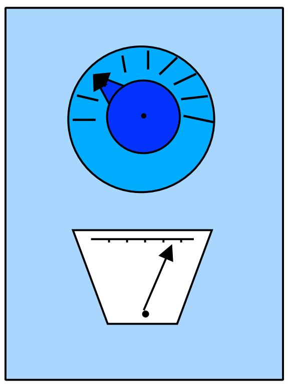
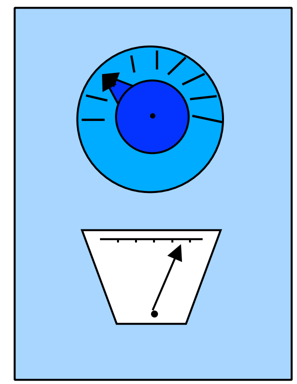
1. Warm-up: CHSH game
2. Bell locality
3. Quantum nonlocality, Bell’s theorem
4. Experiments
5. Loopholes
6. Relevance for device-independent protocols

CORRELATIONS

ALICE (Geneva)



BOB (Bristol)



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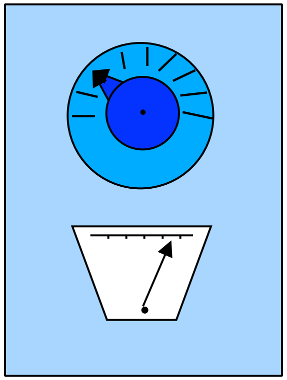

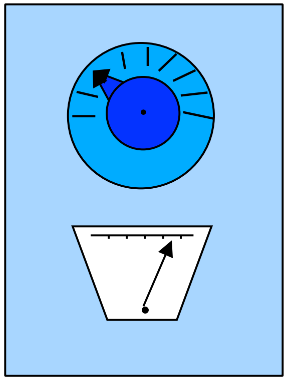

CORRELATED BEHAVIOUR

CORRELATIONS

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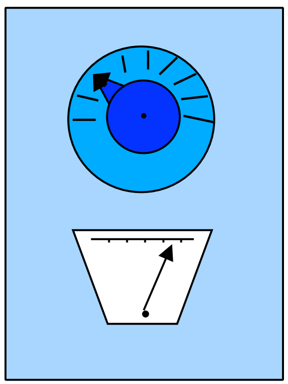

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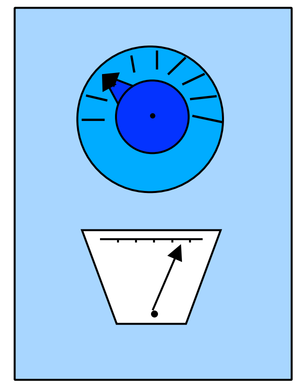
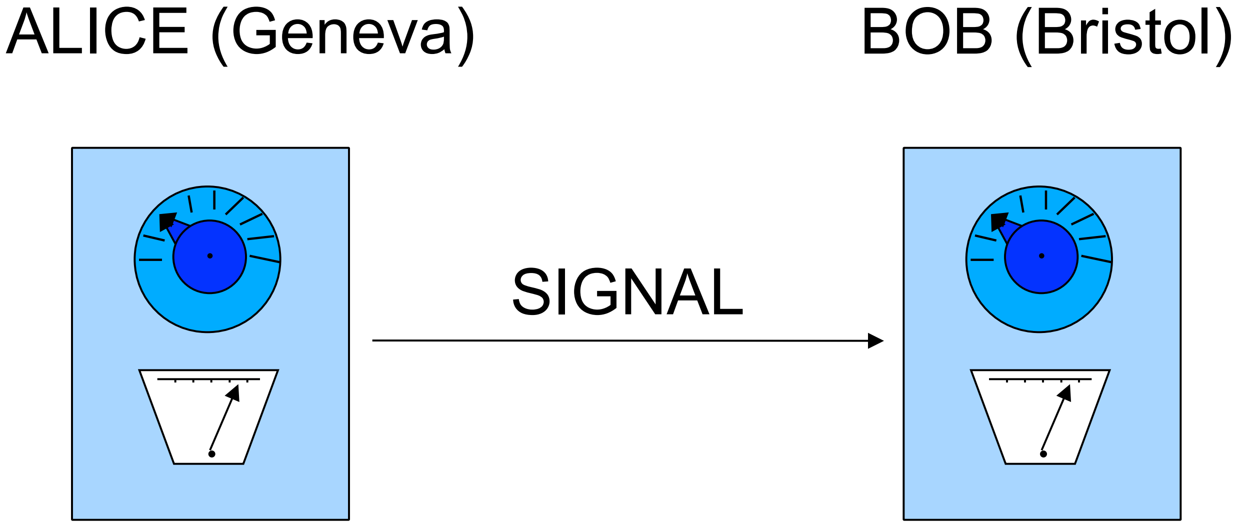


CORRELATED BEHAVIOUR

HOW DOES IT WORK?

CLASSICAL CORRELATIONS



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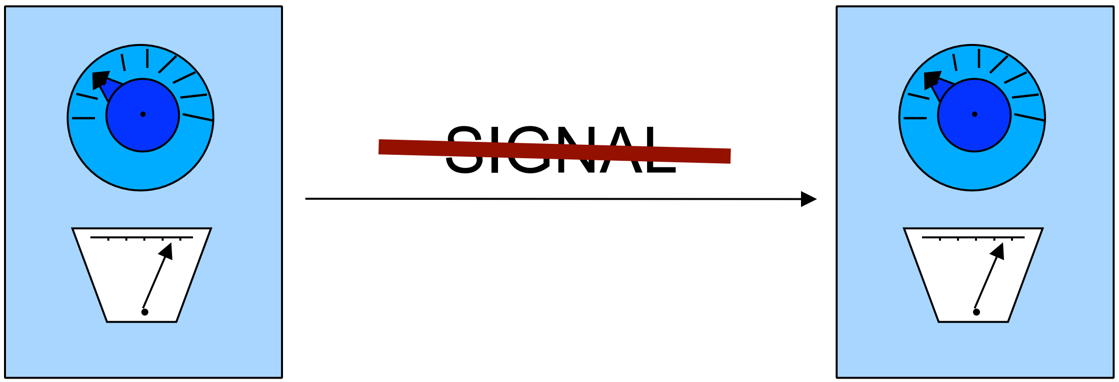

SPACE-LIKE SEPARATION → **NO SIGNAL**

CLASSICAL CORRELATIONS

ALICE (Geneva)

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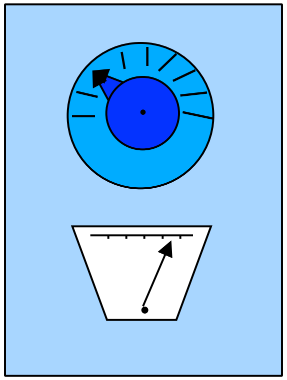

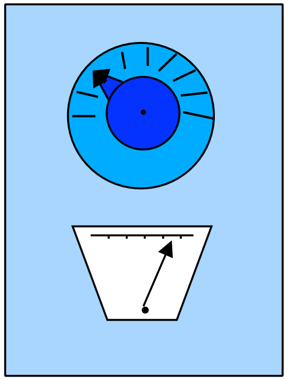

DEVICES HAVE A COMMON **STRATEGY**

PRE-ESTABLISHED CORRELATIONS

CLASSICAL CORRELATIONS

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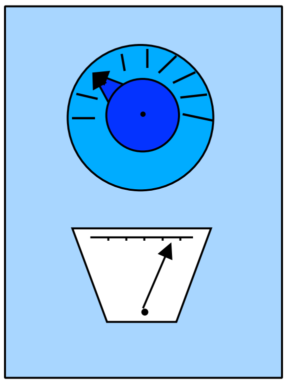

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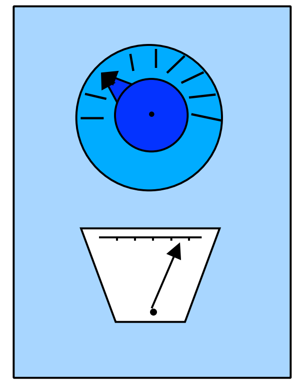
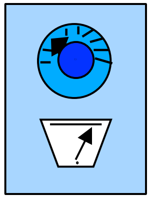
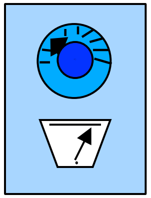


DEVICES HAVE A COMMON **STRATEGY**

PRE-ESTABLISHED CORRELATIONS

→ **CAN THIS BE TESTED?**

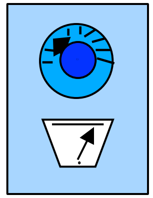
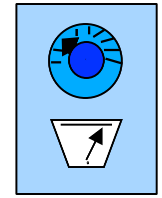
GAME – BELL INEQUALITY

ALICE  BOB 

2 questions: X_0 or X_1 (Alice) Y_0 or Y_1 (Bob)
2 answers: +1 or -1

Bell 64,Clauser-Horne-Shimony-Holt 69

GAME – BELL INEQUALITY

ALICE  BOB 

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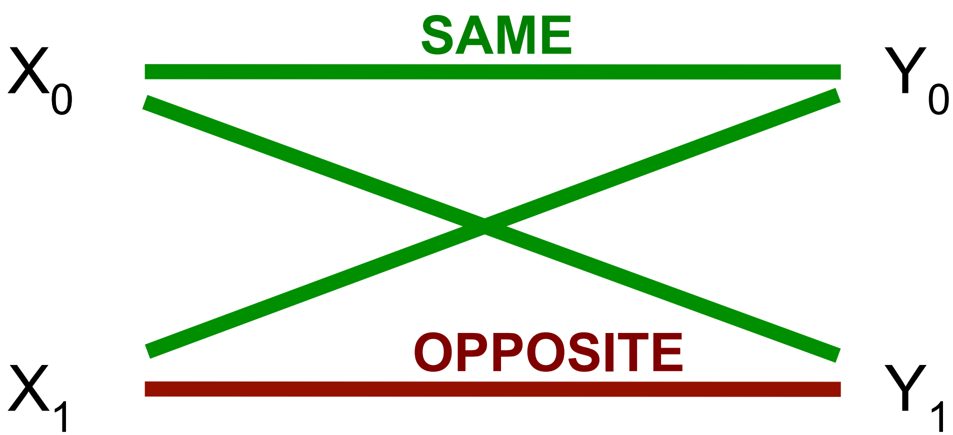

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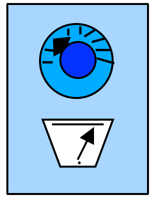
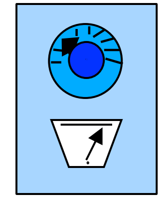
ALICE  BOB 

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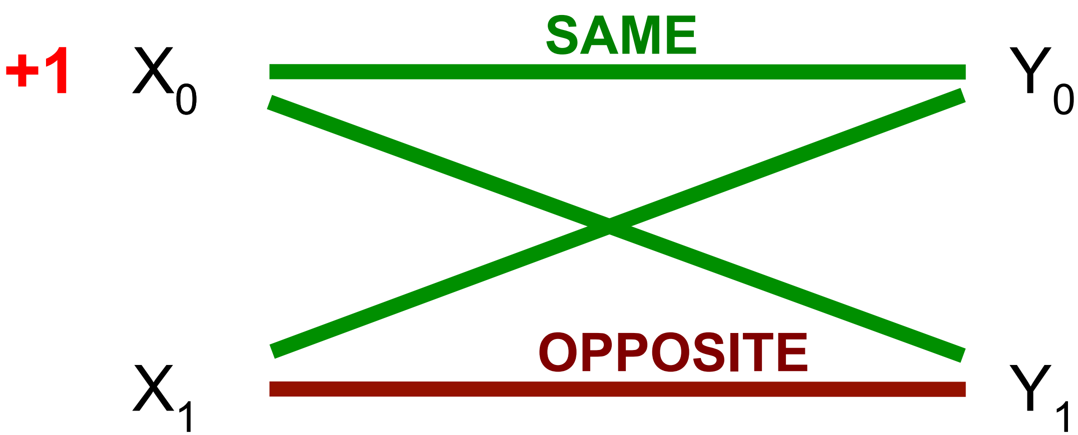

Bell 64, Clauser-Horne-Shimony-Holt 69

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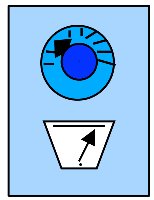

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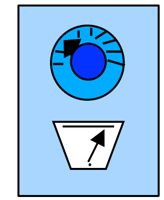

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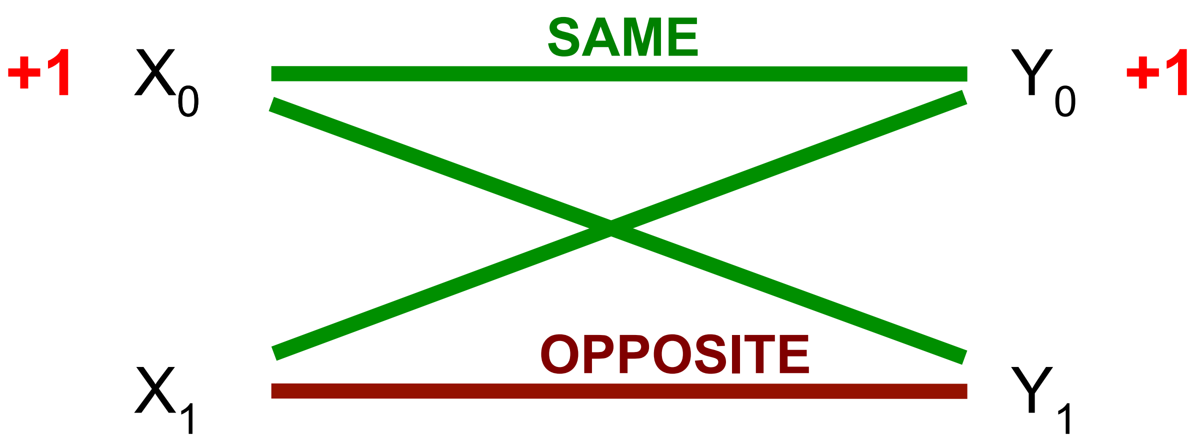

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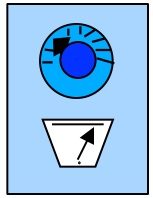
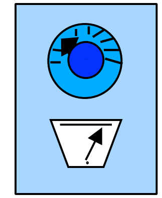
ALICE 

BOB 

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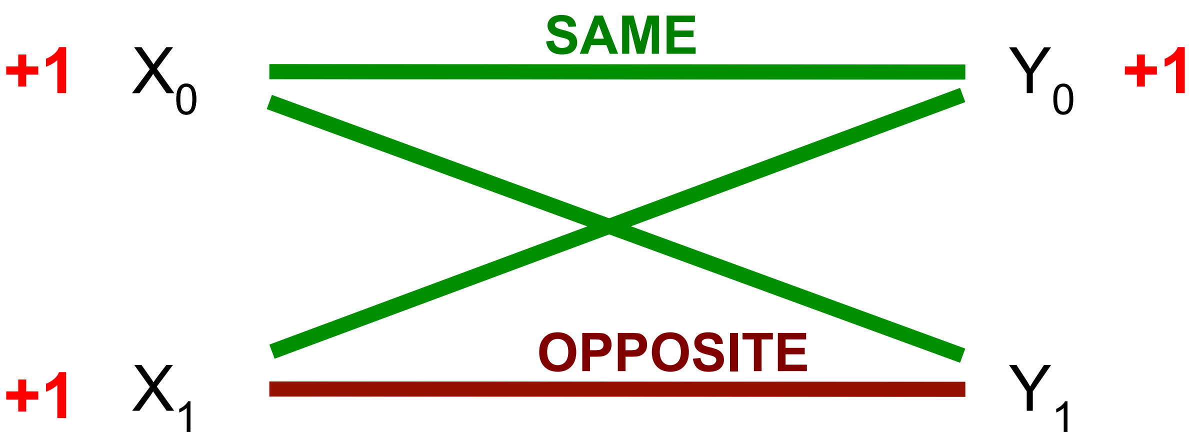

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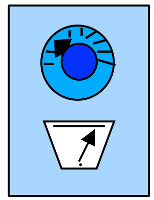
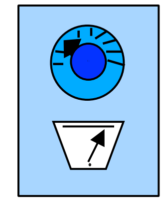
ALICE  BOB 

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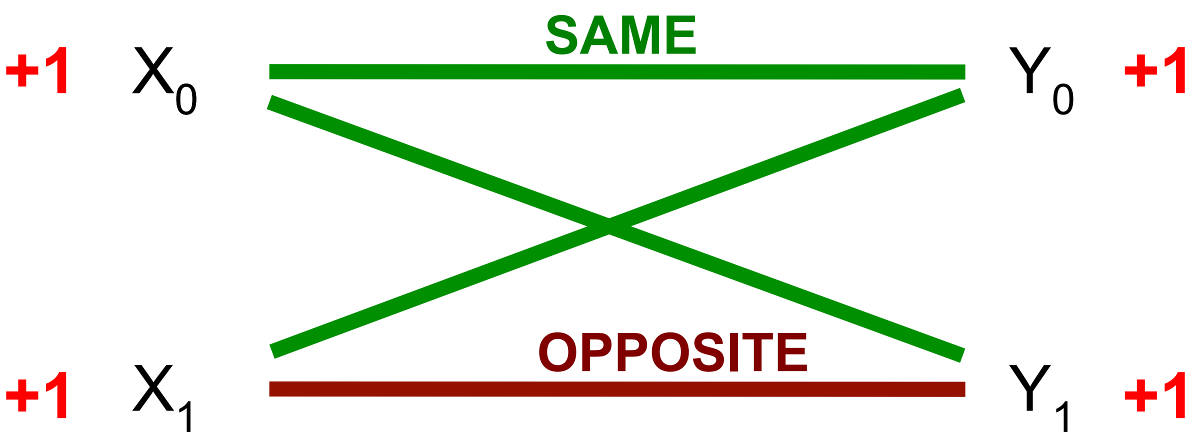

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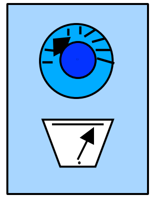
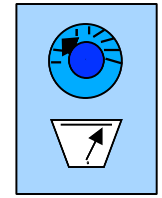
ALICE  BOB 

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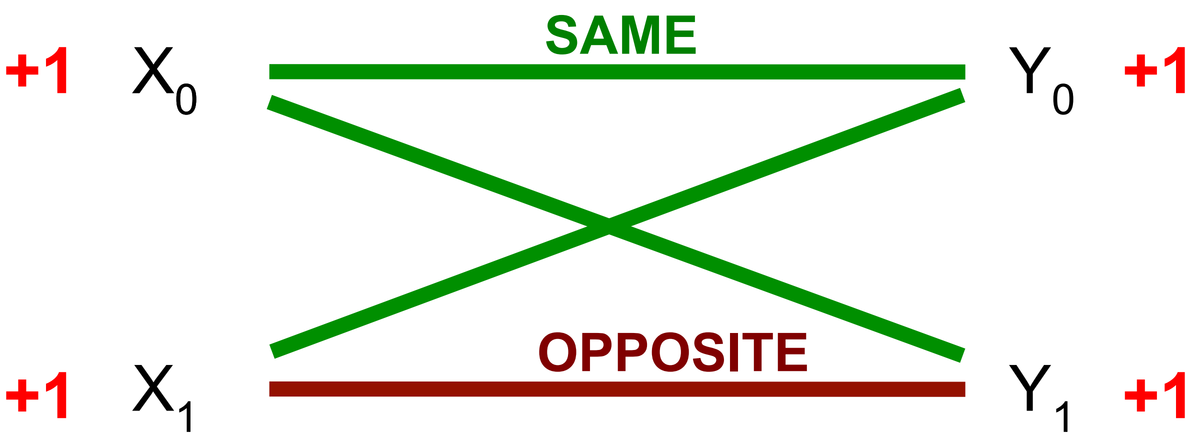

Bell 64,Clauser-Horne-Shimony-Holt 69

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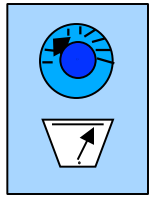

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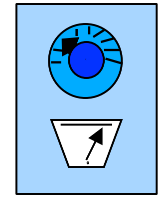

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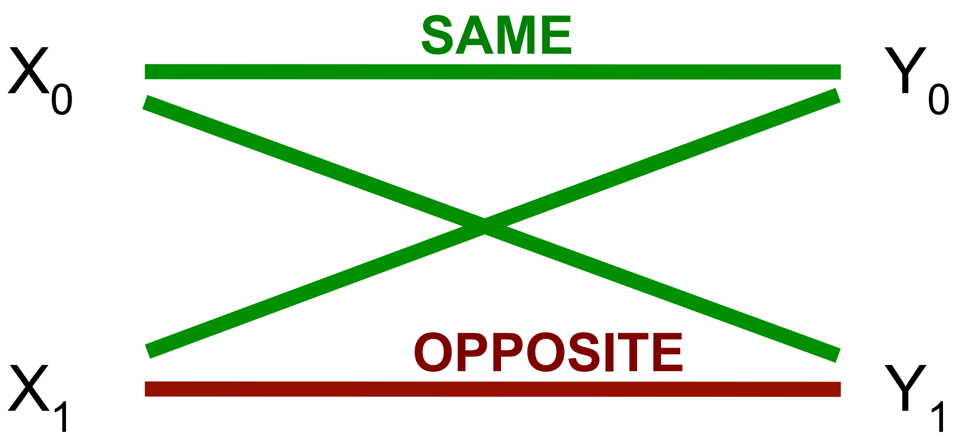
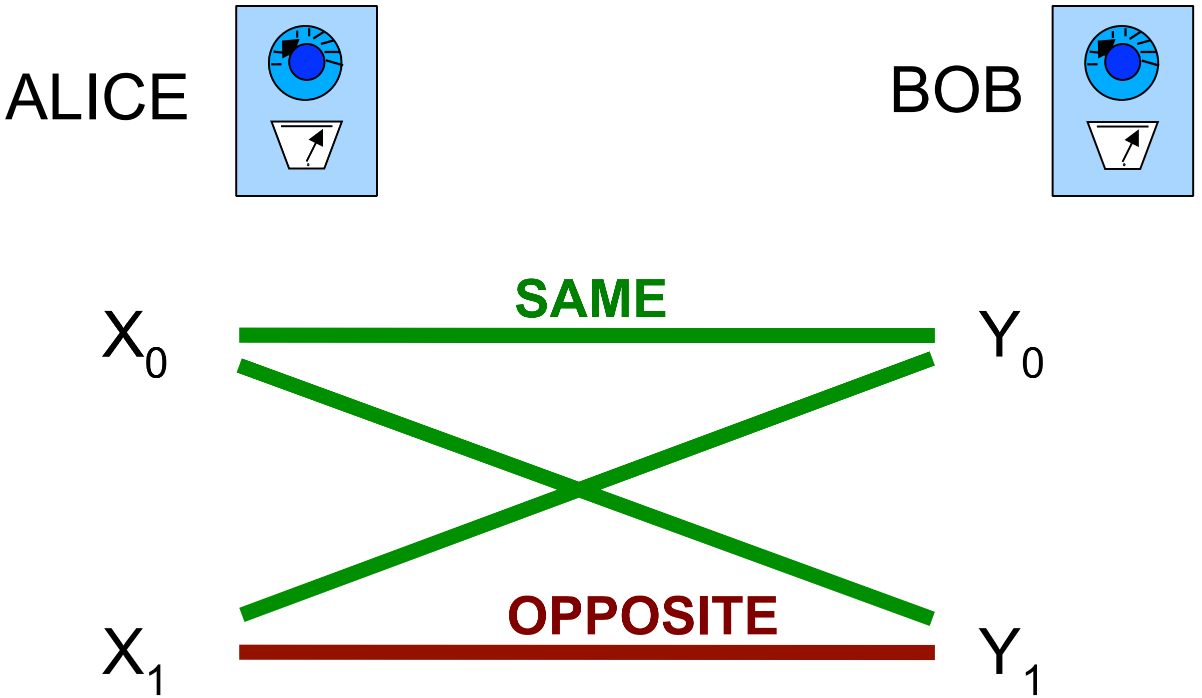


Score ≤ ¾ for ANY classical strategy

Bell 64,Clauser-Horne-Shimony-Holt 69

CHSH BELL INEQUALITY

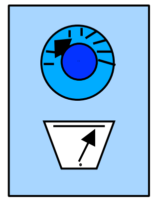


Correlation function: $E(X_0,Y_1) = p(X_0=Y_1) - p(X_0 \neq Y_1)$

Clauser-Horne-Shimony-Holt 69

CHSH BELL INEQUALITY

ALICE



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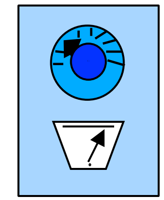

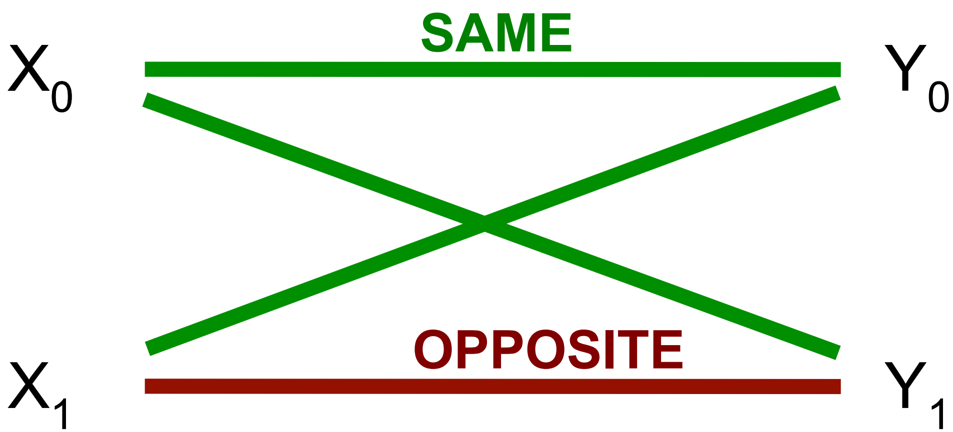

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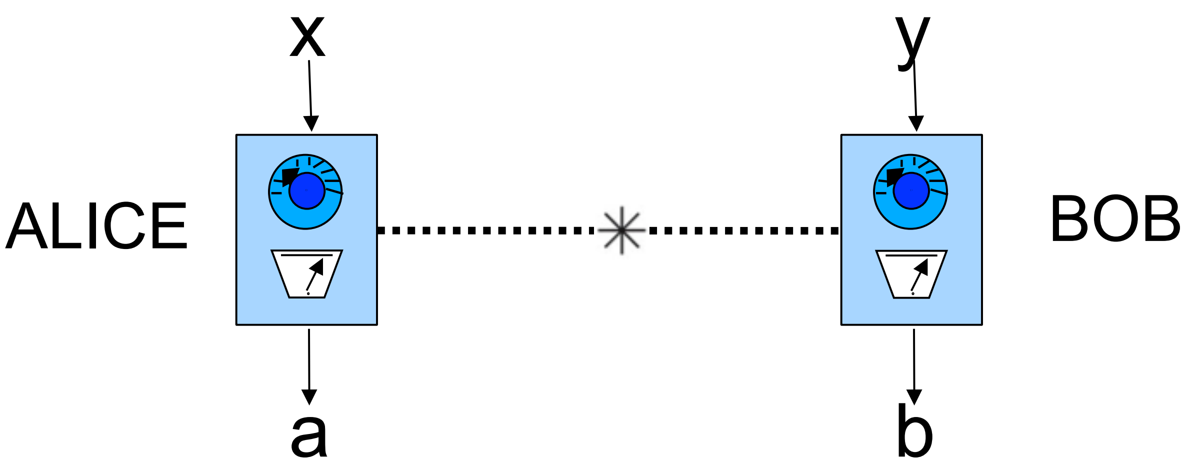
$$CHSH = E(X_0,Y_0) + E(X_0,Y_1) + E(X_1,Y_0) - E(X_1,Y_1) \leq 2$$

Clauser-Horne-Shimony-Holt 69

Locality (à la Bell)



Data: joint prob distribution $p(a,b|x,y)$

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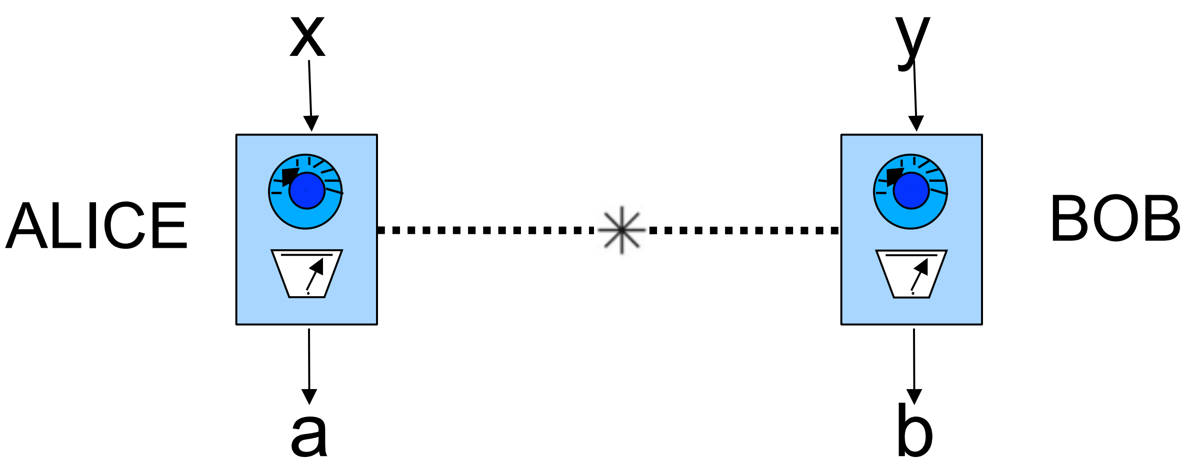

Data: joint prob distribution p(a,b|x,y)

Can this data be explained by a local model?

Bell 64

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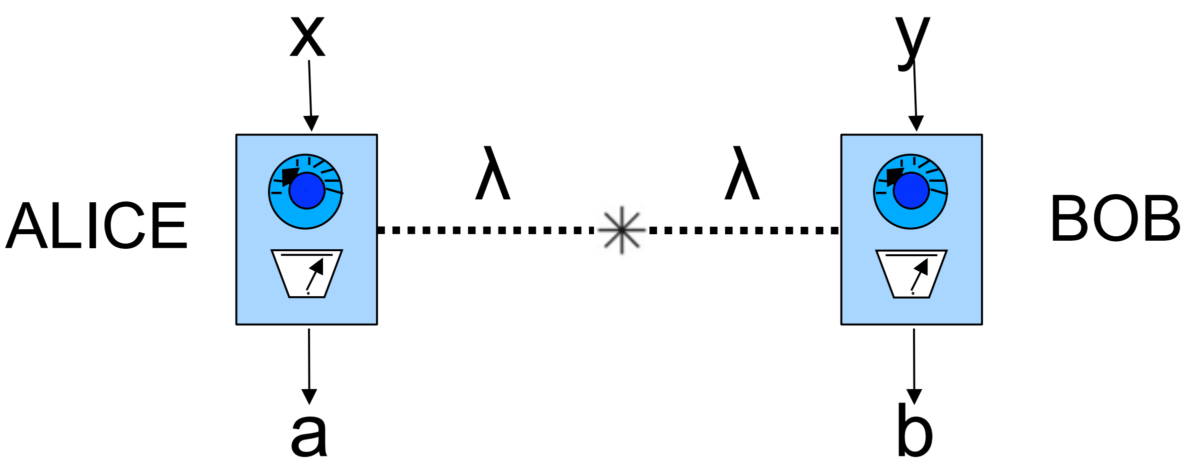


Data: joint prob distribution $p(a,b|x,y)$

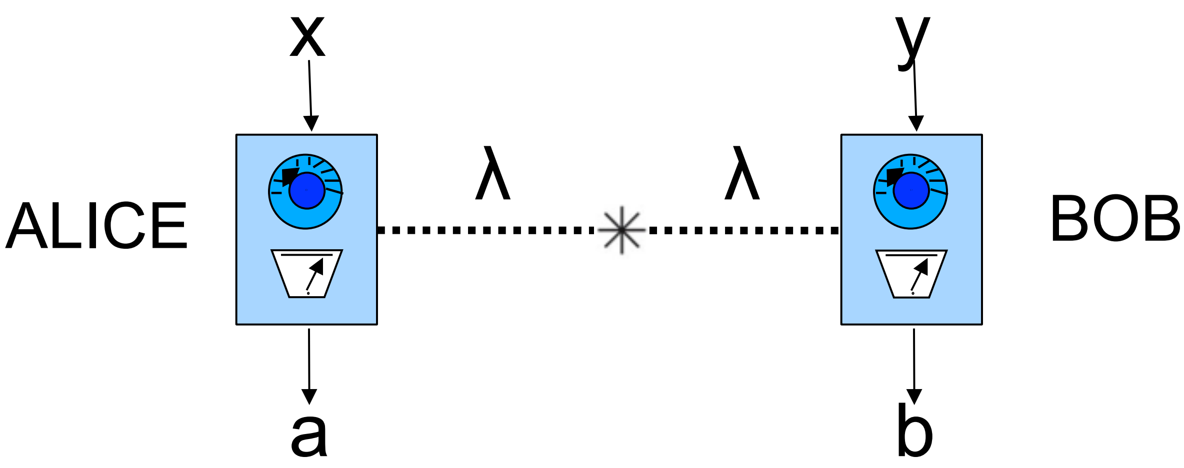
Can this data be explained by a local model?

Bell locality: $p(a,b|x,y) = \int d\lambda\, p(\lambda)\ \ p(a,b|x,y,\lambda)$

$= \int d\lambda\, p(\lambda)\, p(a|x,\lambda)\, p(b|y,\lambda)$

Bell 64

Locality (à la Bell)



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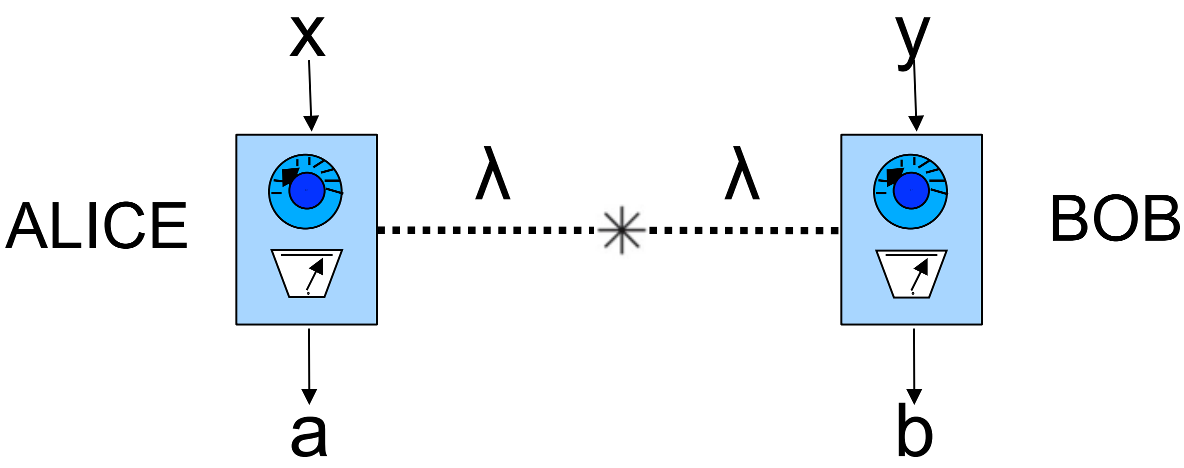

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Local correlations satisfy ALL Bell inequalities

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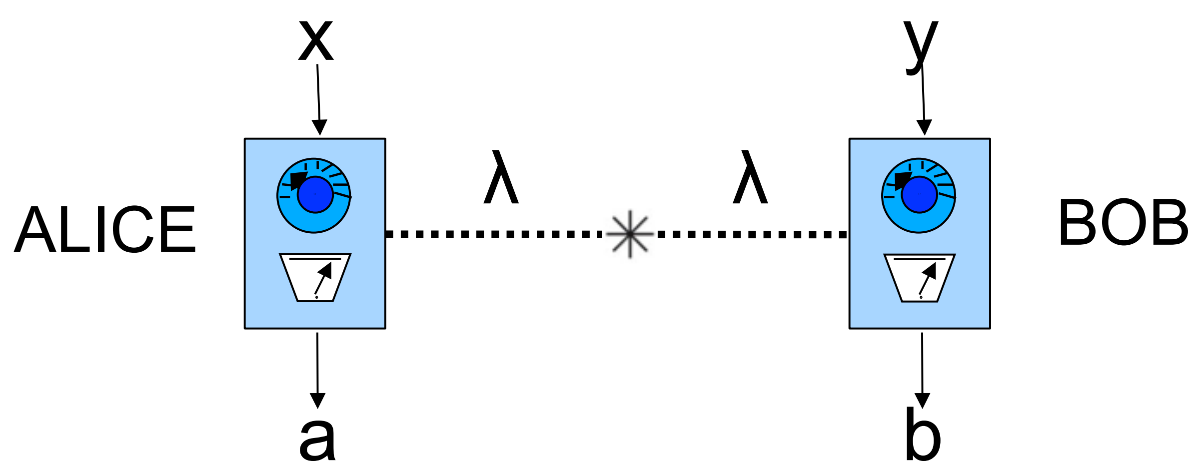


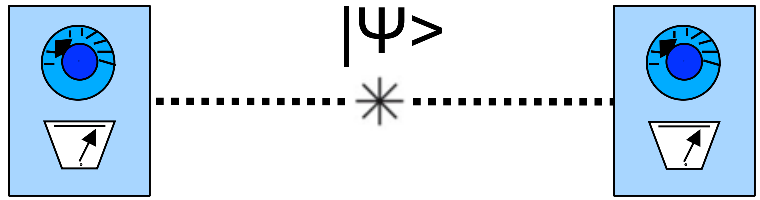
Bell locality : $p(a,b|x,y) = \int d\lambda \; p(a|x,\lambda)\; p(b|y,\lambda)$

Local correlations satisfy ALL Bell inequalities

Violation of a Bell inequality implies **NONLOCALITY**

USING QUANTUM RESOURCES

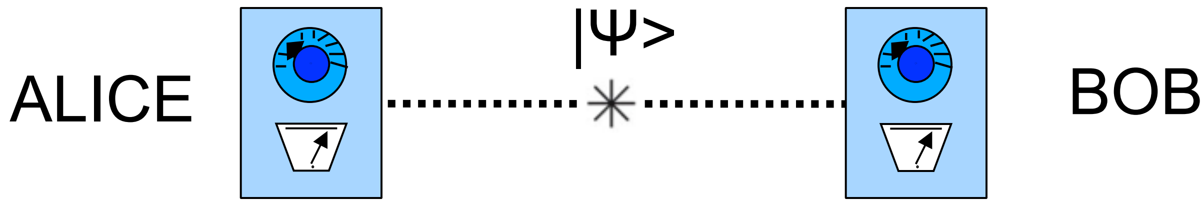
ALICE



BOB

Quantum strategy

USING QUANTUM RESOURCES

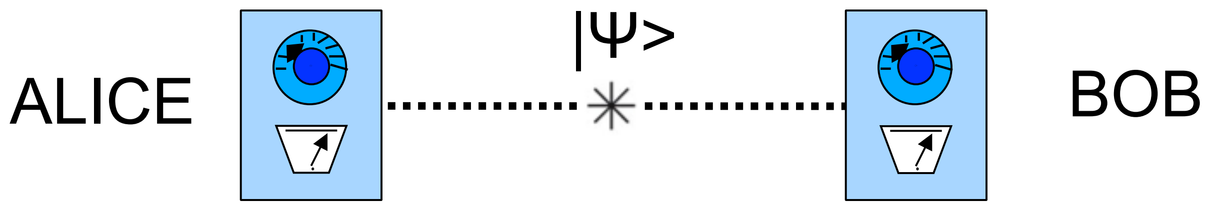


Quantum strategy

1. Entangled state $|\Psi\rangle = |0,1\rangle - |1,0\rangle$

2. Local meas $X_0 = \vec{z}$ $X_1 = \vec{x}$ and $Y_0 = -\vec{x}-\vec{z}$ $Y_1 = \vec{x}-\vec{z}$

USING QUANTUM RESOURCES



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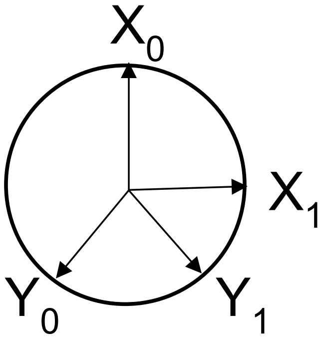

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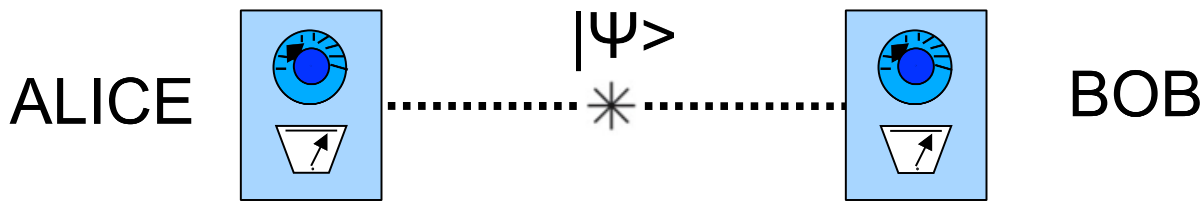

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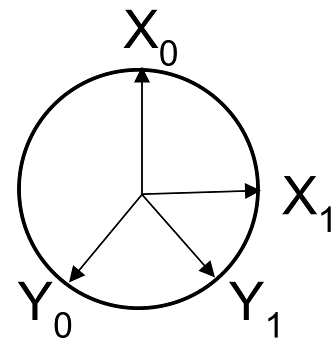
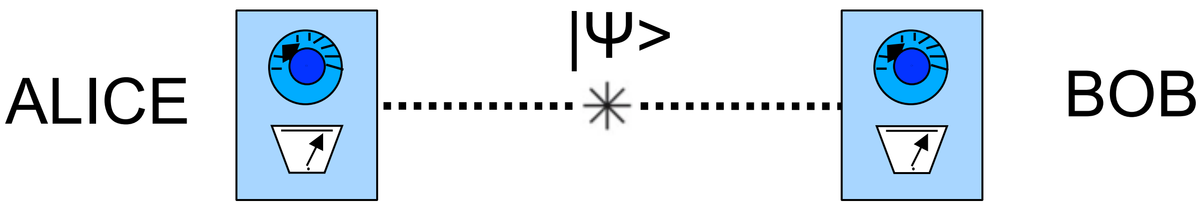


$$CHSH = E(X_0,Y_0) + E(X_0,Y_1) + E(X_1,Y_0) - E(X_1,Y_1)$$

USING QUANTUM RESOURCES



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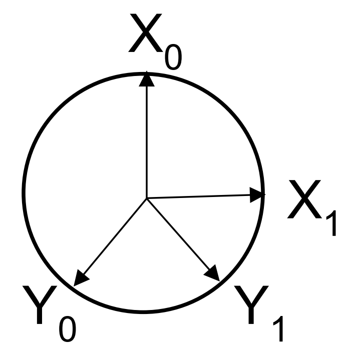
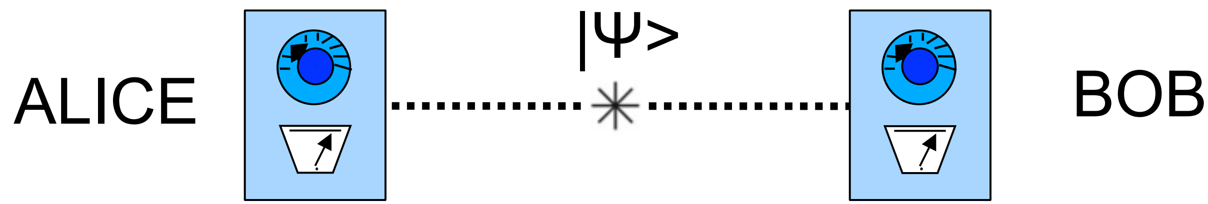


$$\text{CHSH} = E(X_0,Y_0) + E(X_0,Y_1) + E(X_1,Y_0) - E(X_1,Y_1)$$

$$= 1/\sqrt{2}$$

USING QUANTUM RESOURCES



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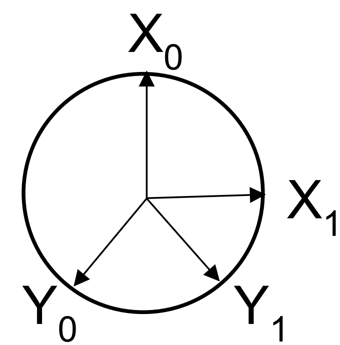

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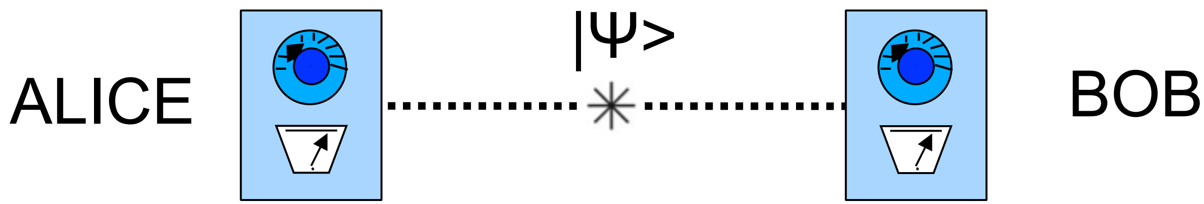

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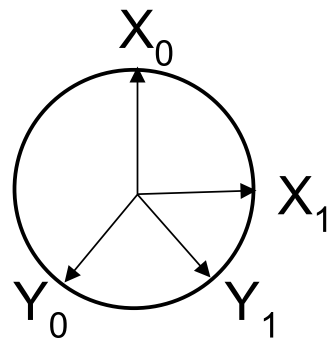
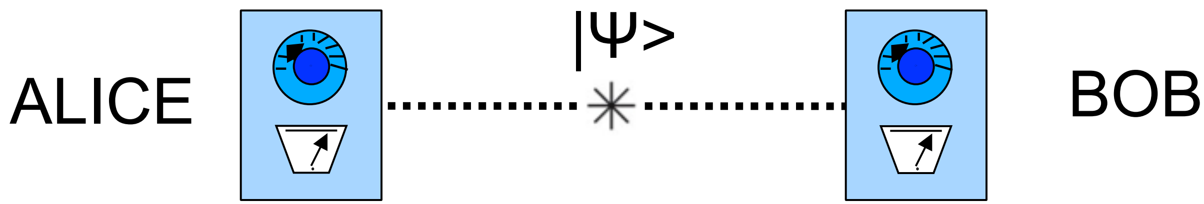
$$E(\vec{a},\vec{b}) = \langle\Psi|\, \vec{a}\ \vec{b}\, |\Psi\rangle = -\, \vec{a}\ \vec{b}$$



$$CHSH = E(X_0,Y_0) + E(X_0,Y_1) + E(X_1,Y_0) - E(X_1,Y_1)$$

$= 1/\sqrt{2}$

USING QUANTUM RESOURCES



Quantum strategy

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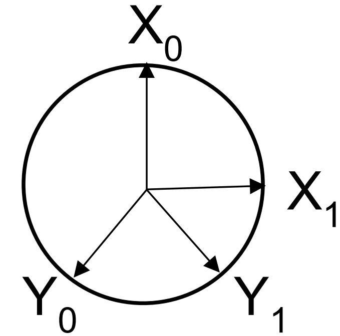

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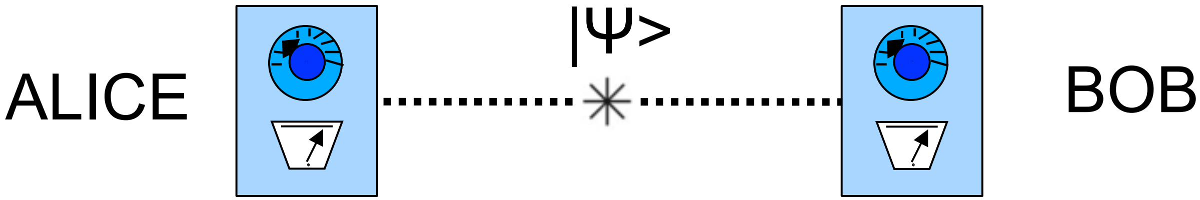
$$= -1/\sqrt{2}$$

USING QUANTUM RESOURCES



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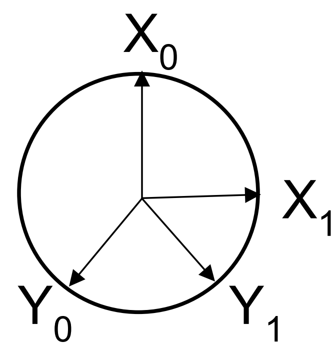
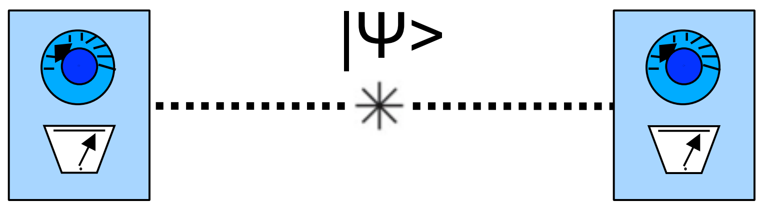


$$CHSH = E(X_0,Y_0) + E(X_0,Y_1) + E(X_1,Y_0) - E(X_1,Y_1) = 2\sqrt{2} > 2$$

USING QUANTUM RESOURCES

ALICE  BOB

Quantum strategy

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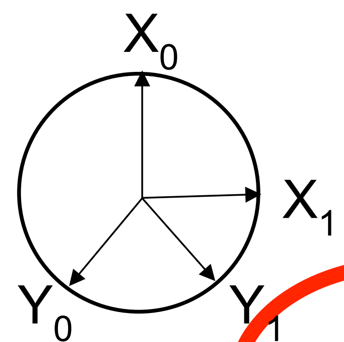

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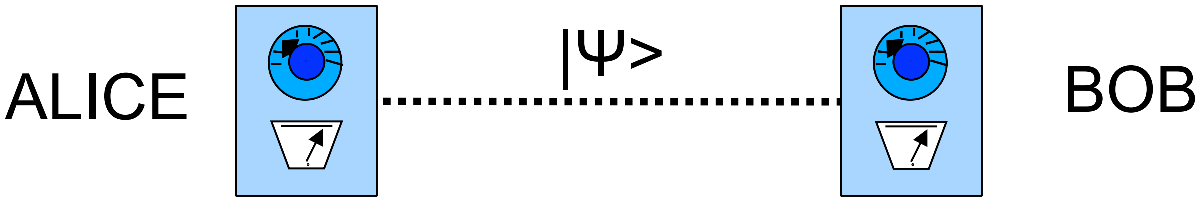
QUANTUM NONLOCALITY

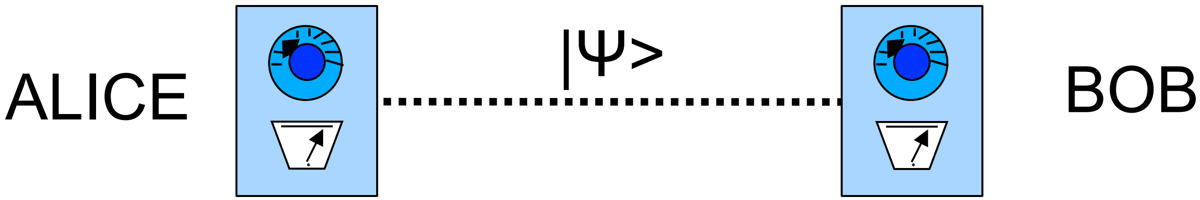
BELL'S THEOREM



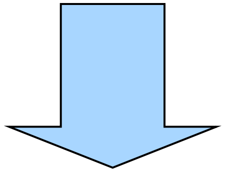
Quantum correlations are NONLOCAL

Stronger than **ANY** local correlations

BELL’S THEOREM



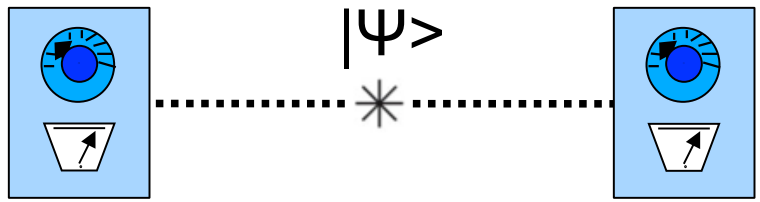
Quantum correlations are NONLOCAL



Stronger than **ANY** local correlations

<u>Bell’s Theorem:</u> Predictions of QM are incompatible with ANY theory satisfying Bell locality

BEST QUANTUM STRATEGY

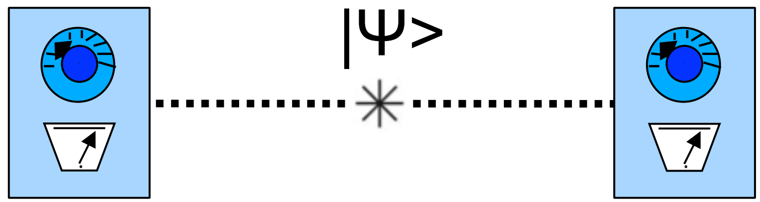
ALICE  BOB

$$\text{CHSH} \leq 2 + \| 2 [X_0,X_1] [Y_0,Y_1] \|^{1/2} \leq 2\sqrt{2}$$

Tsirelson’s bound

BEST QUANTUM STRATEGY

ALICE



BOB

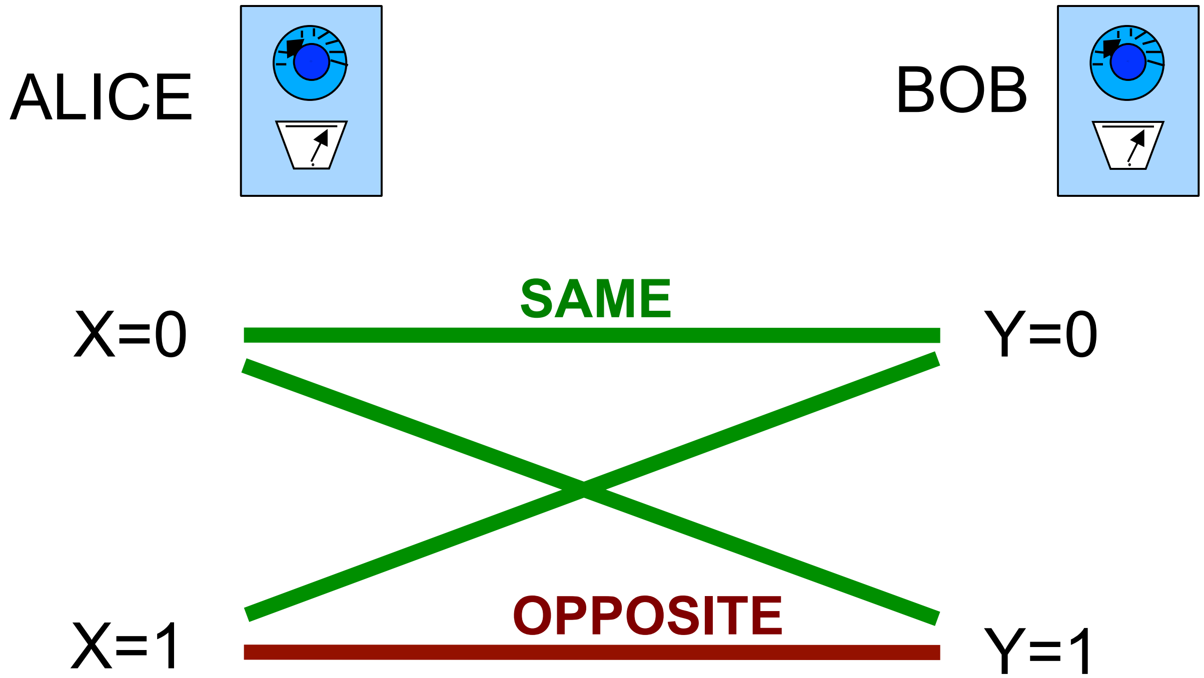
$$\text{CHSH} \leq 2 + \| 2\,[X_0,X_1]\,[Y_0,Y_1] \|^{1/2} \leq 2\sqrt{2}$$

Tsirelson's bound

Bell violation $\Rightarrow$

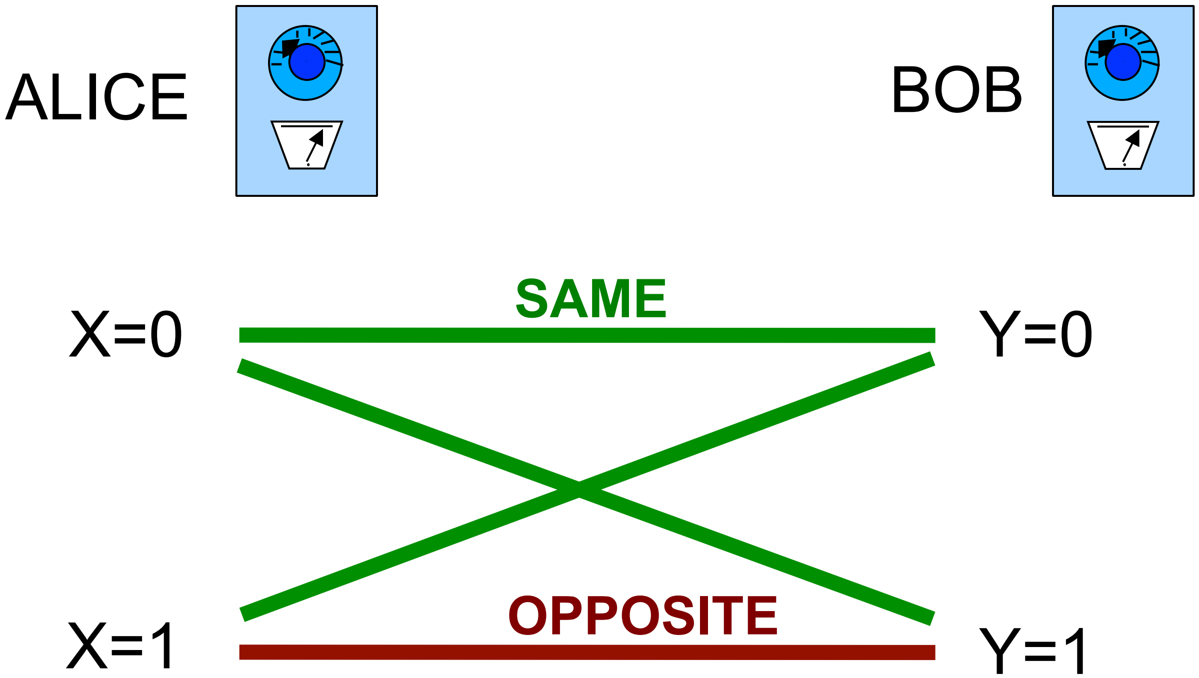
1. Entanglement
2. Incompatible measurements

BEYOND QM



Best possible score?

BEYOND QM

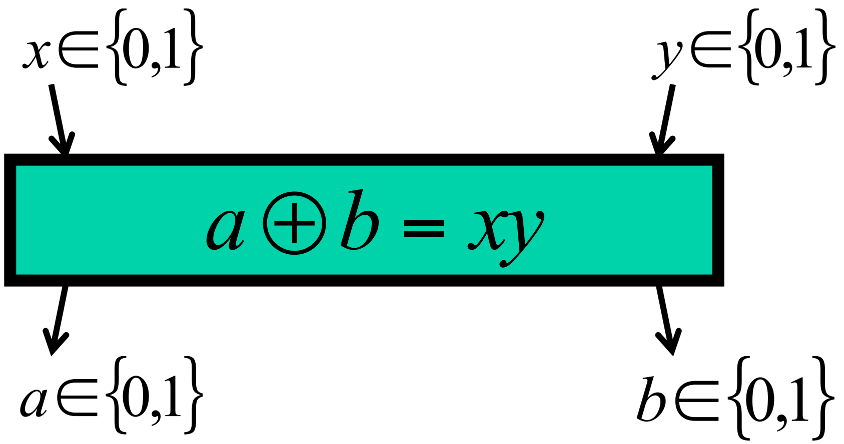


Best possible score?

$$\text{CHSH} = E(X=Y=0) + E(X=0,Y=1) + E(X=1,Y=0) - E(X=Y=1) = 4$$

Can this be reached?

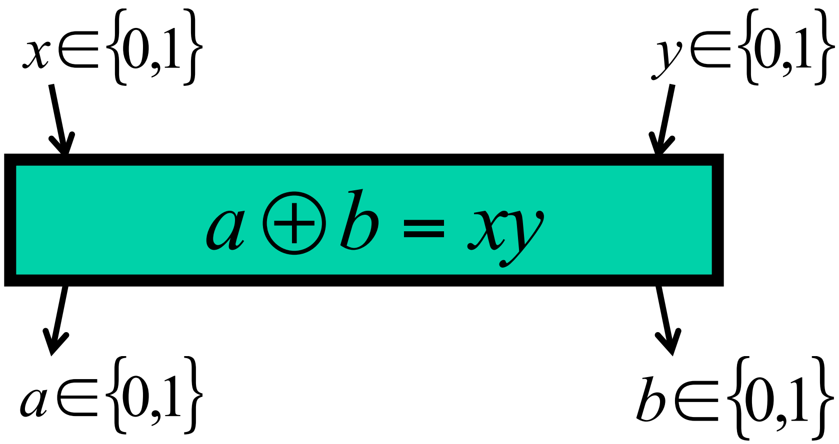
PR BOX



- Non-signaling
- Maximally nonlocal CHSH = 4

Popescu-Rohrlich 1994, Review: Popescu 2014

PR BOX



- Non-signaling
- Maximally nonlocal CHSH = 4

WHY DOES THE PR BOX NOT EXIST IN NATURE ?

Popescu-Rohrlich 1994, Review: Popescu 2014

EXPERIMENTS

EXPERIMENTS / LOOPHOLES

Technical imperfections open loopholes

EXPERIMENTS / LOOPHOLES

Technical imperfections open loopholes

1. **LOCALITY LOOPHOLE**

→ Space-like separation

EXPERIMENTS / LOOPHOLES

Technical imperfections open loopholes

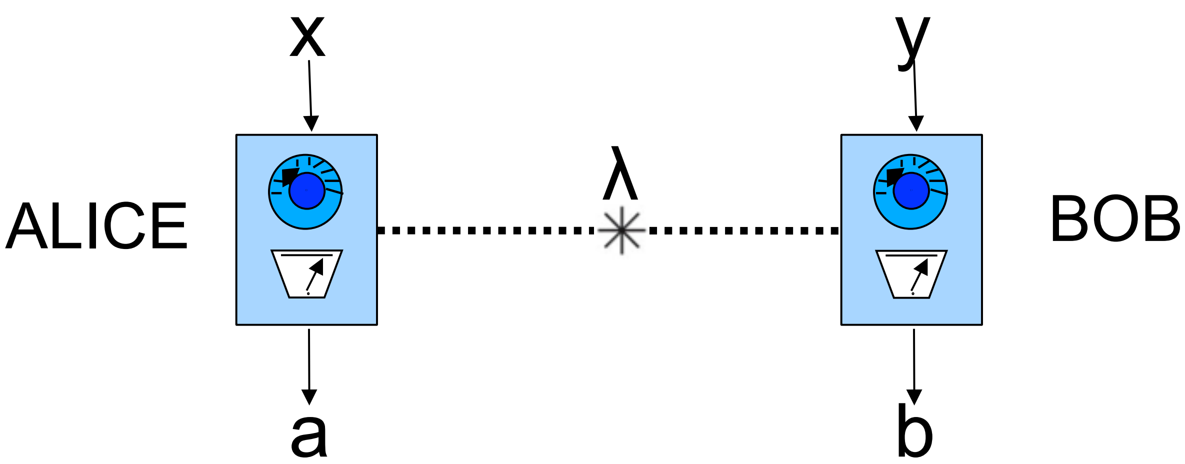
1. **LOCALITY LOOPHOLE**

→ Space-like separation

2. **DETECTION LOOPHOLE**

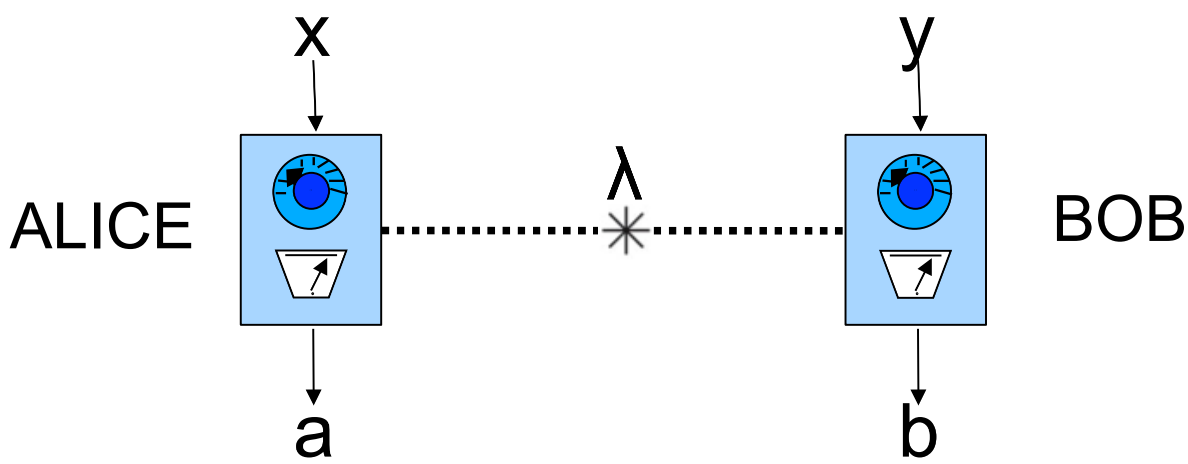
→ High detection efficiency

Locality loophole



Bell locality**:** $p(a,b|x,y) = \int d\lambda\, p(\lambda)\, p(a|x,\lambda)\, p(b|y,\lambda)$

Locality loophole



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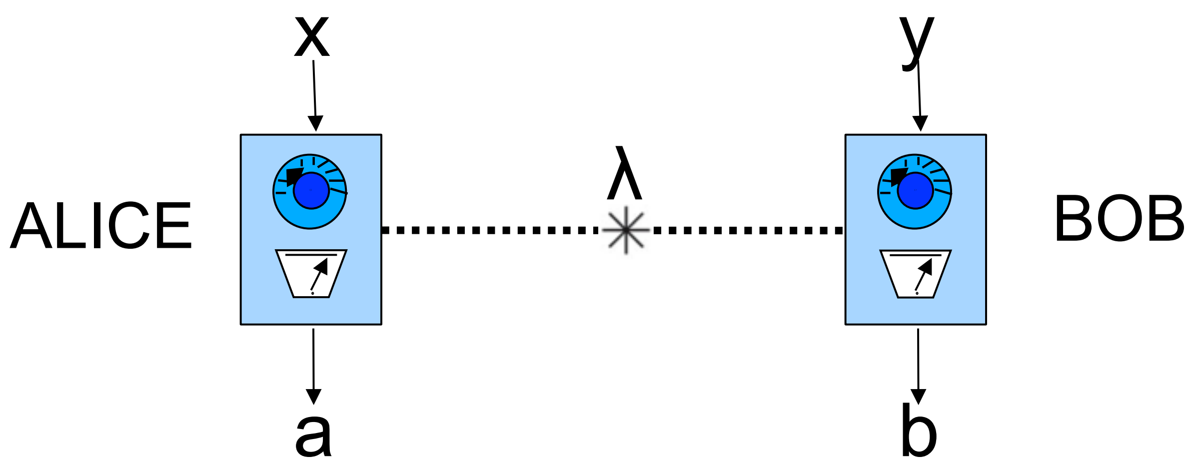
No communication between Alice and Bob

$p(a|x,y,b,\lambda) = p(a|x,\lambda)$ & $p(b|x,y,a,\lambda) = p(b|y,\lambda)$

Locality loophole



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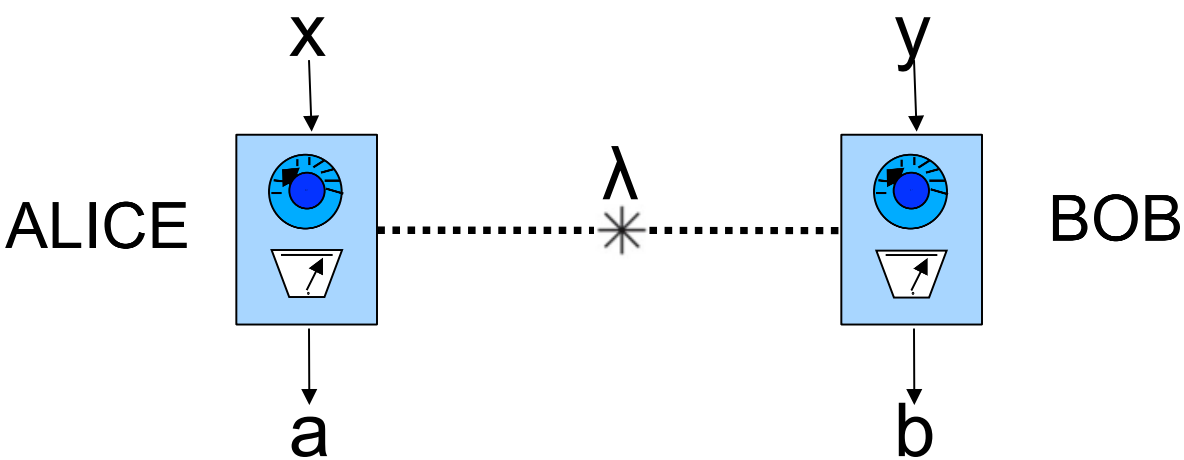
No communication between Alice and Bob

$p(a|x,y,b,\lambda) = p(a|x,\lambda)$ & $p(b|x,y,a,\lambda) = p(b|y,\lambda)$

⇒ **Space-like separation must be enforced**

Locality loophole

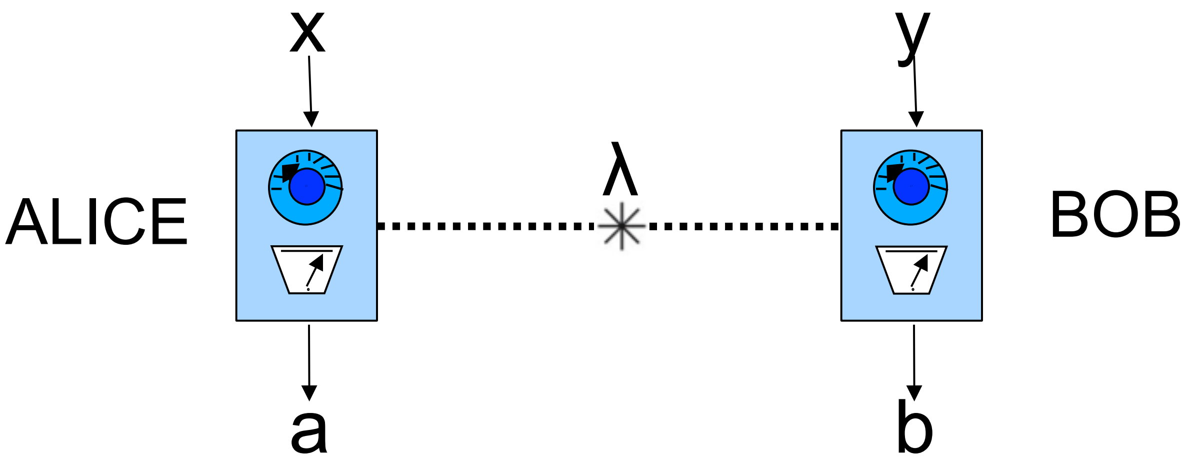


Bell locality: p(a,b|x,y) = ∫ dλ p(λ) p(a|x,λ) p(b|y,λ)

Random (or free) choice of settings x,y

p(λ|x,y) = p(λ)

Locality loophole



Bell locality**:** $p(a,b|x,y) = \int d\lambda \, p(\lambda) \, p(a|x,\lambda) \, p(b|y,\lambda)$

Random (or free) choice of settings x,y

$p(\lambda|x,y) = p(\lambda)$

RNGs space-like separated from source

Photonic experiments / locality loophole

1972 Freedman & Clauser

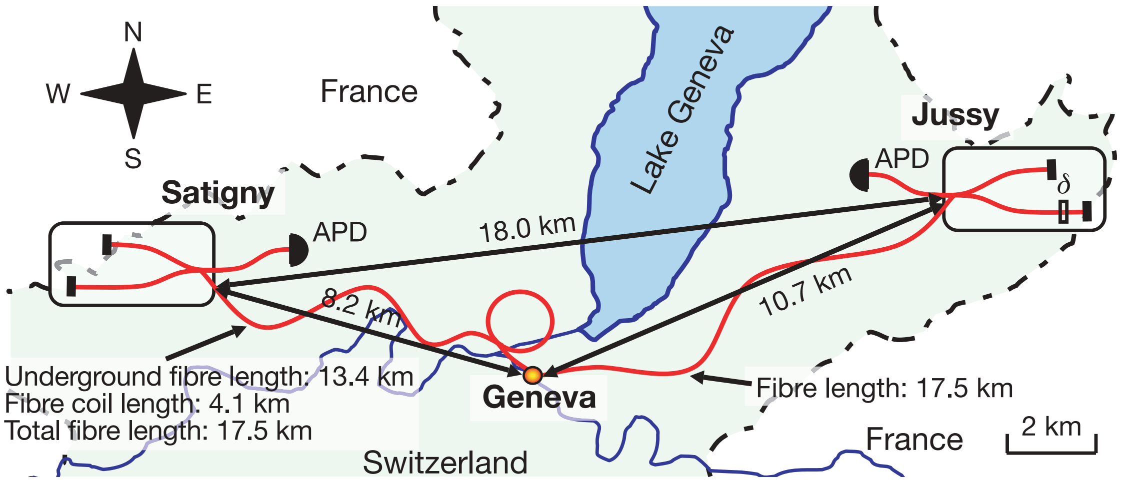
Closing locality loophole

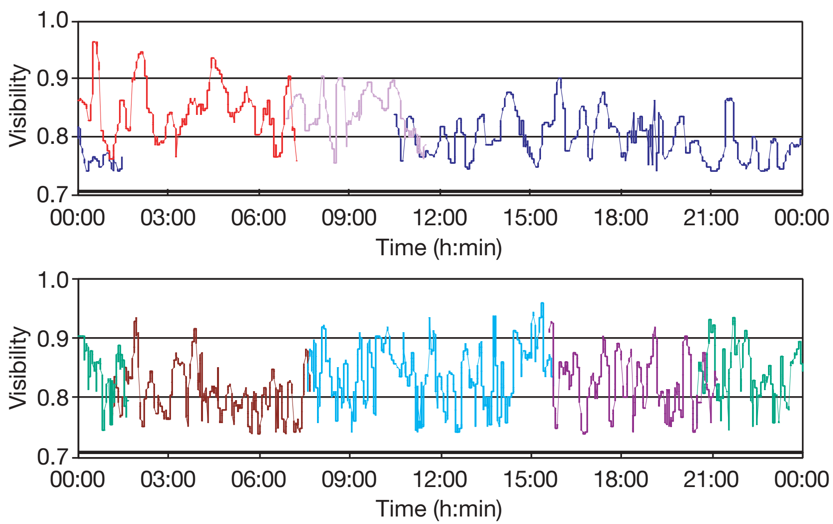
1982 Aspect, Dalibard, Roger

1998 Tittel et al. 10km
Weihs et al. Einstein locality

2010 Scheidl et al. PNAS 'Freedom of choice' loophole

SPOOKY ACTION AT A DISTANCE ?





No visibility drop for 24 hours

→ Speed of signal > 10^5 c

Salart et al. Nature 2008

Detection loophole

Idea: if the (observed) detection efficiency is too low, a local model can lead to Bell inequality violation

Pearle PRD 1970

Detection loophole

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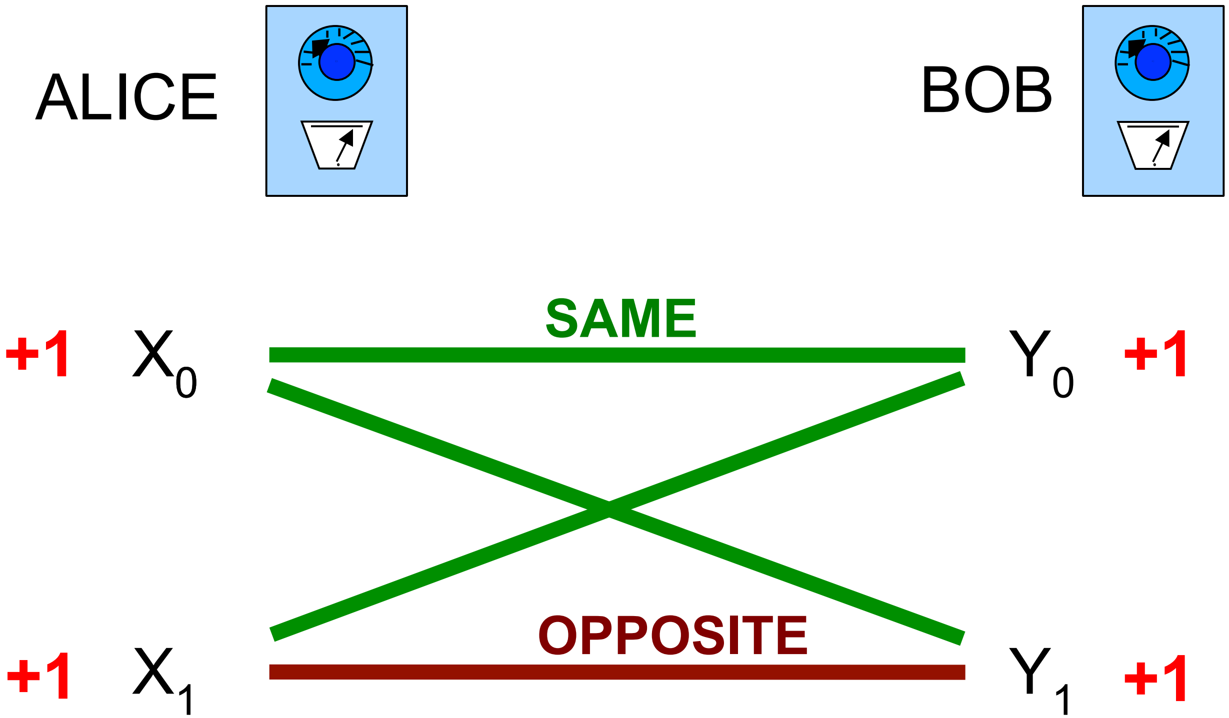
Pearle PRD 1970

Treshold efficiency is typically high (75%)

Example: CHSH



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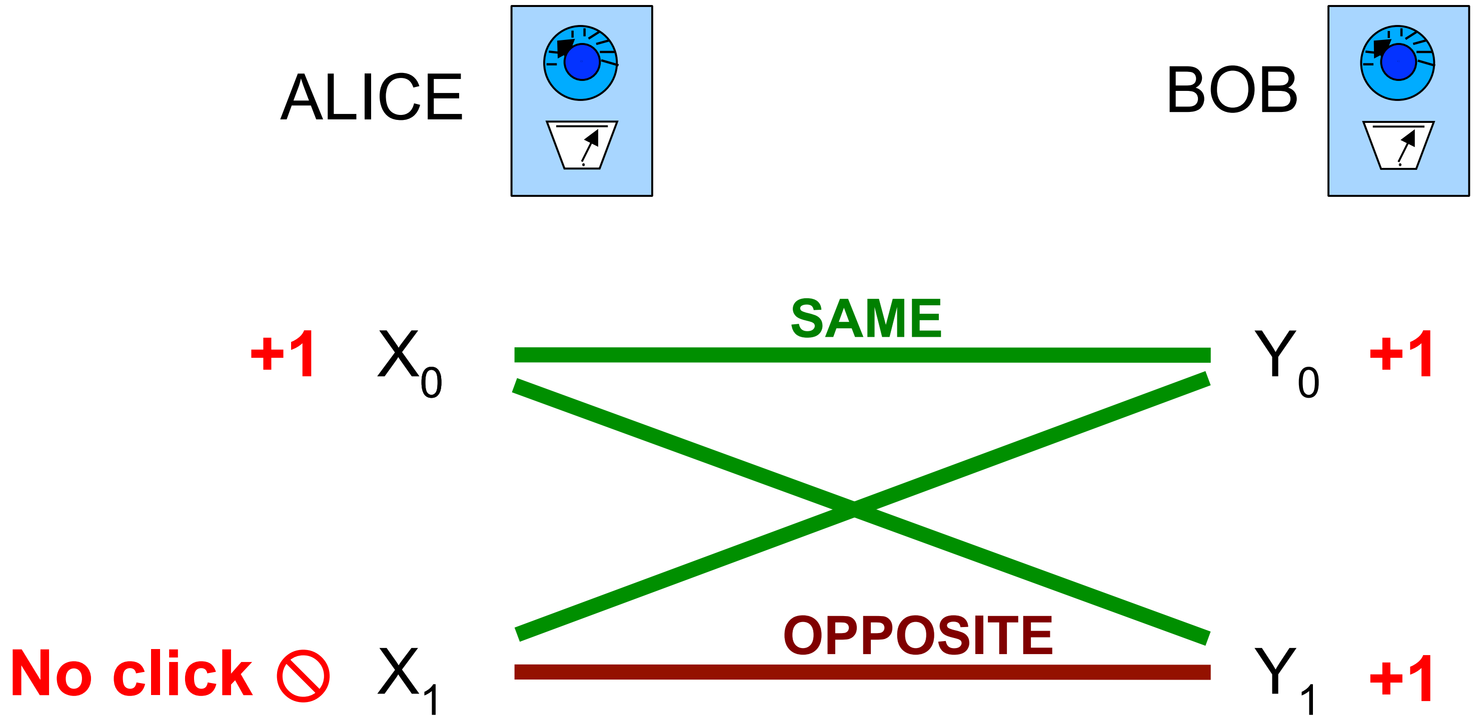

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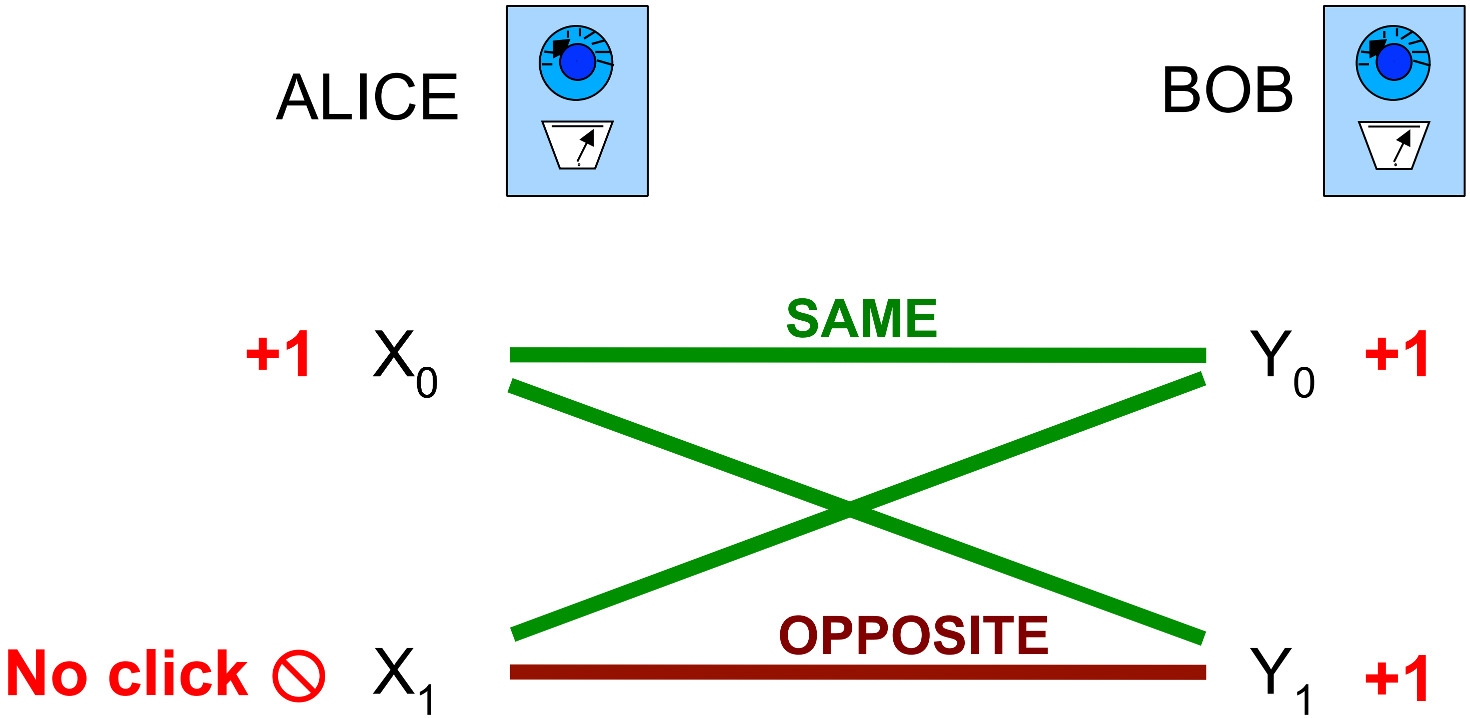
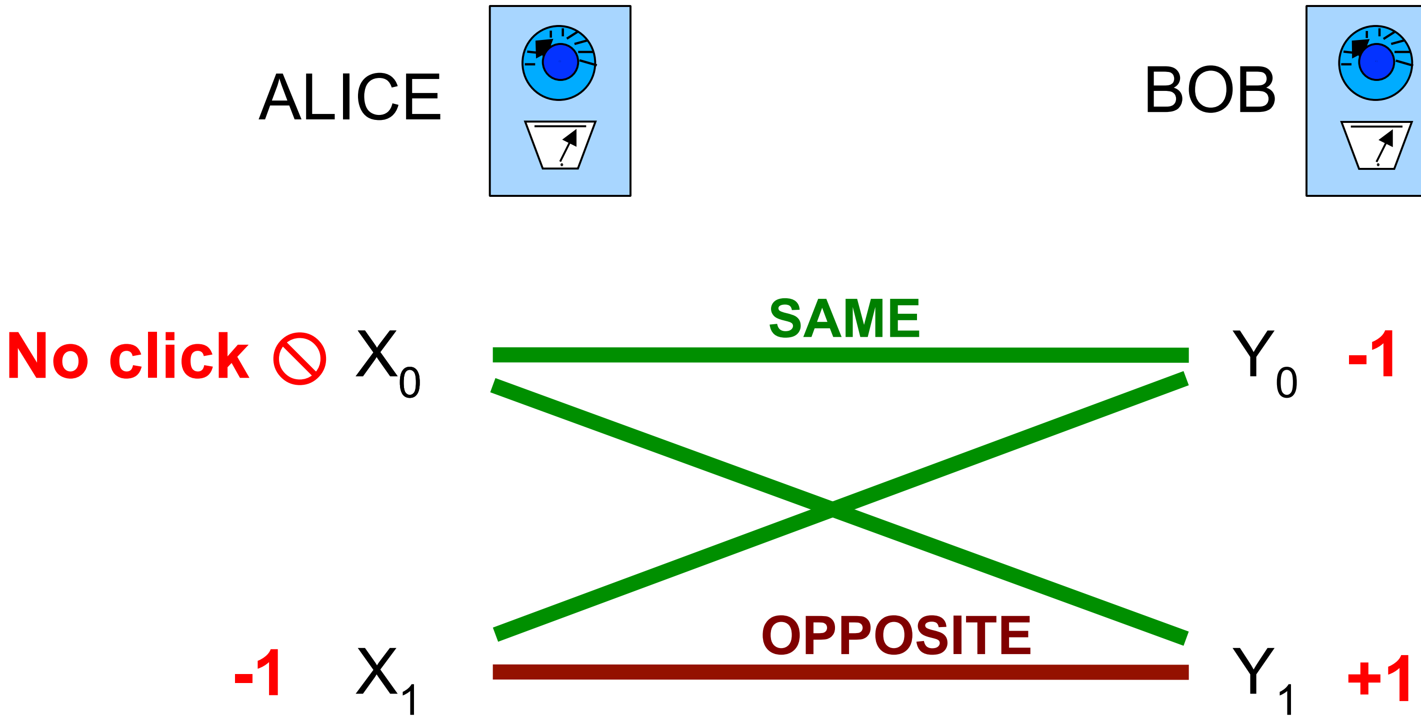


Strategy exploiting the possibility of not (always) answering

Example: CHSH



Strategy exploiting the possibility of not (always) answering

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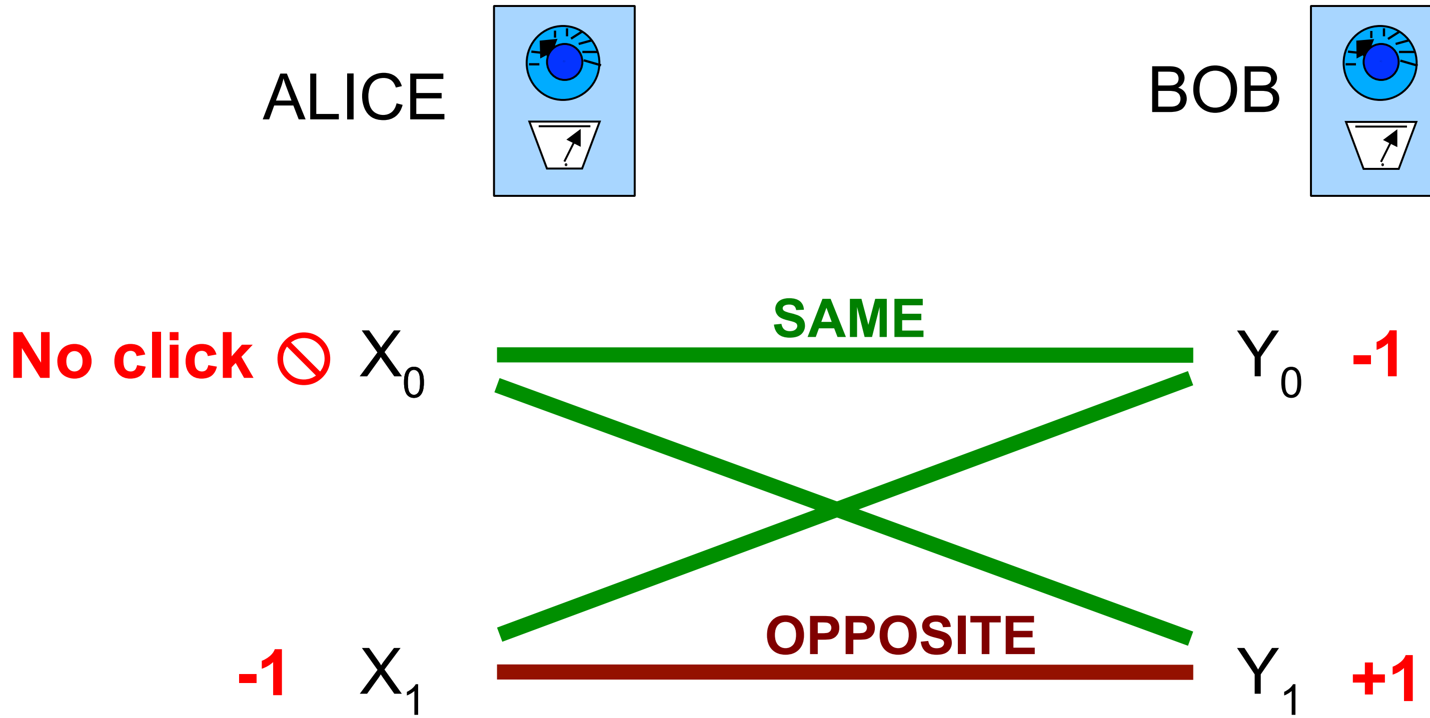
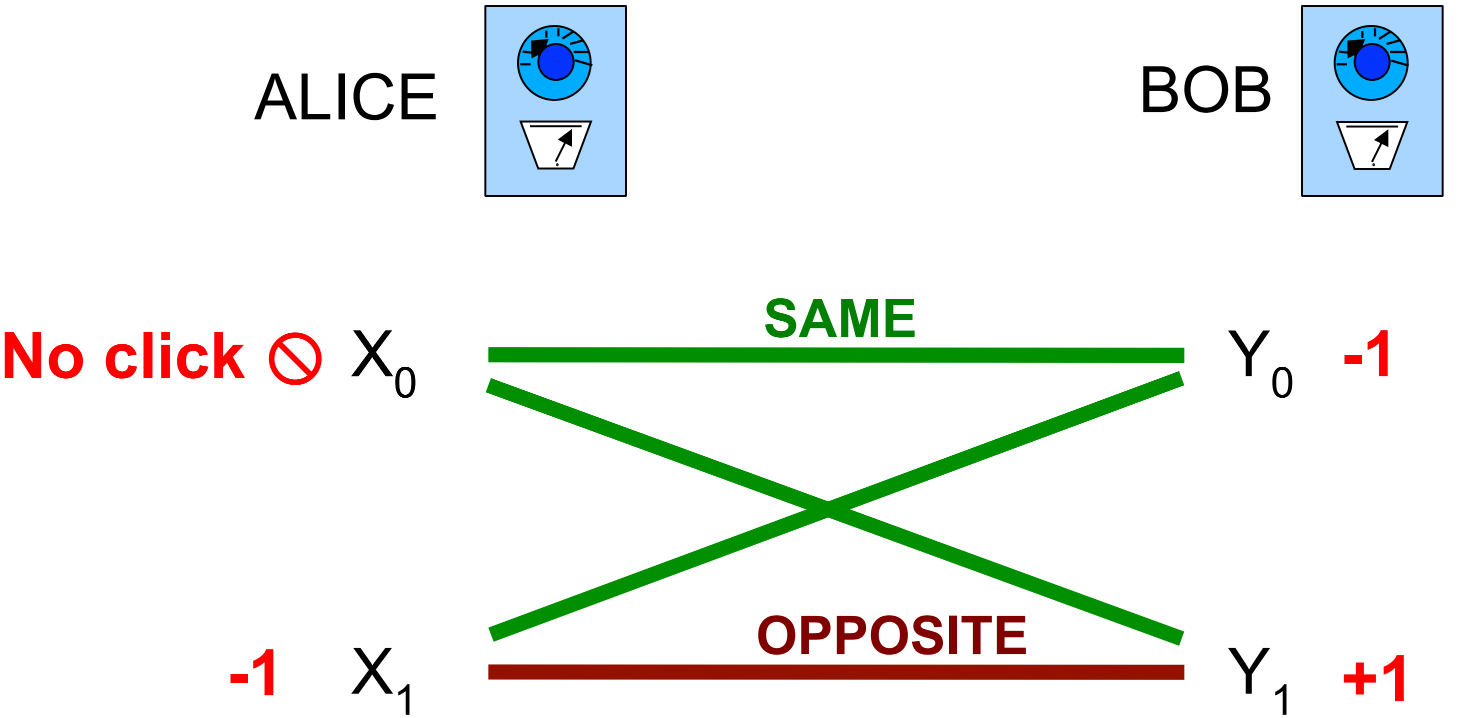


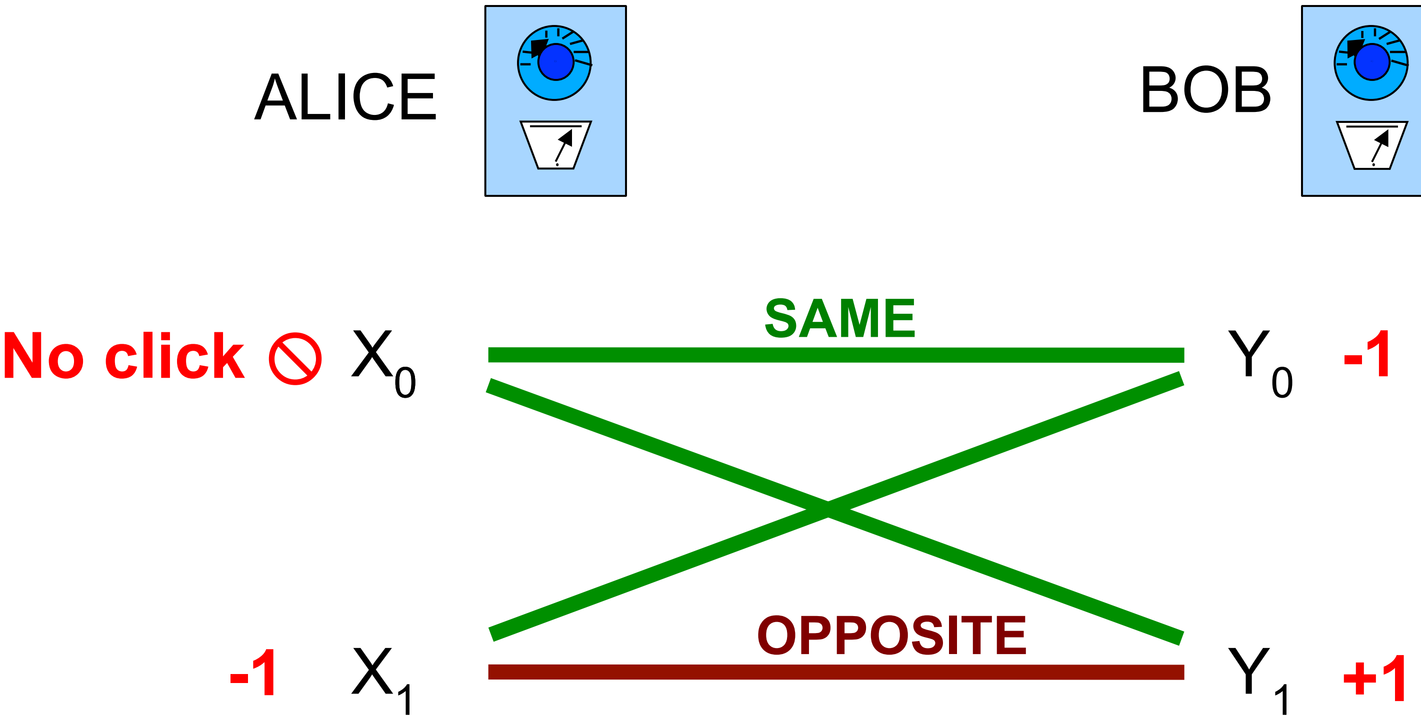
Alice's detector clicks with prob ½

Example: CHSH



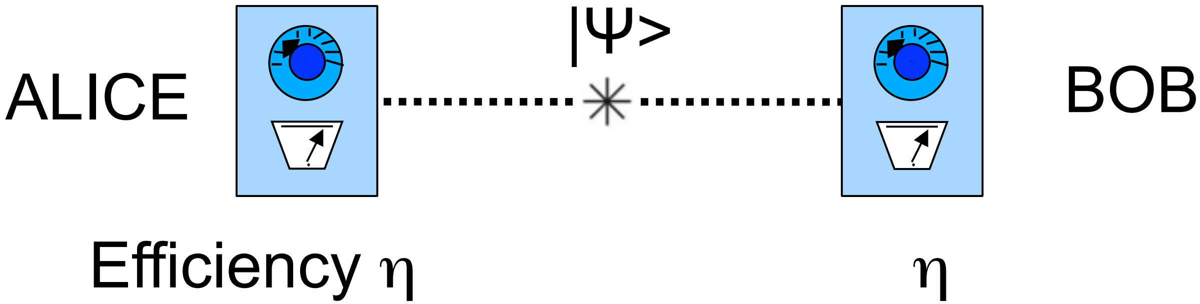
- Alice's detector clicks with prob ½
- Post-selected statistics gives CHSH = 4

Example: CHSH



Local model (exploiting detection loophole) reproducing PR box

Threshold efficiency for CHSH



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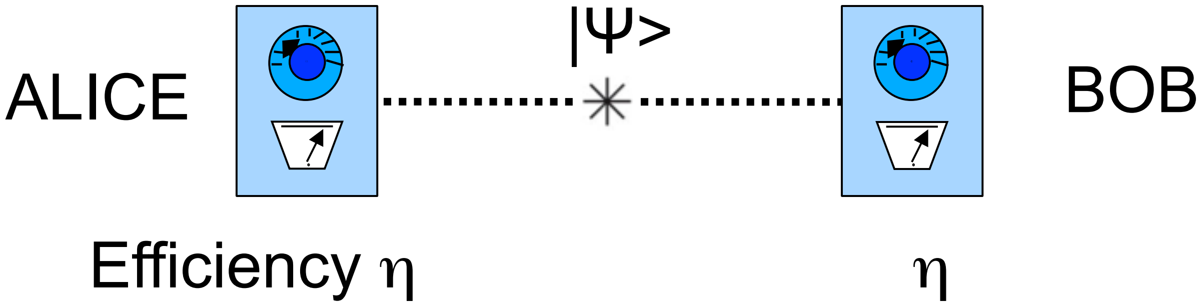
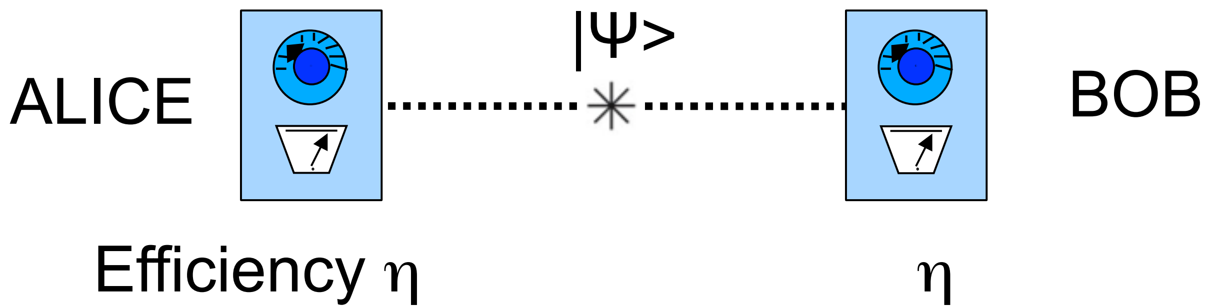


Minimal efficiency required?

Threshold efficiency for CHSH



Minimal efficiency required?

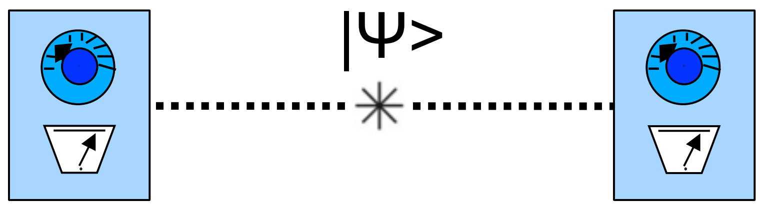
Full statistics violates CHSH $\eta^2 2\sqrt{2} + (1-\eta)^2 2 > 2.$

Clicks A & B

No click A & B

Threshold efficiency for CHSH

ALICE



BOB

Efficiency η η

Minimal efficiency required?

Full statistics violates CHSH $\eta^2 2\sqrt{2} + (1-\eta)^2 2 > 2.$

Clicks A & B No click A & B

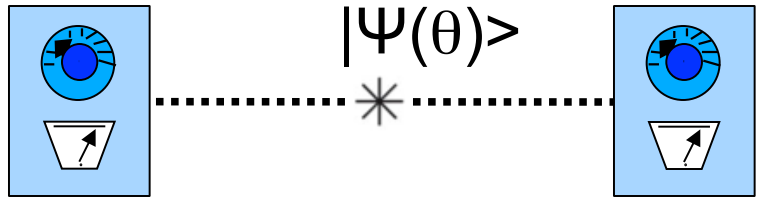
$$\eta > \eta^* = \frac{2}{1+\sqrt{2}} \approx 82.8\%.$$

Minimal efficiency using two-qubit singlet

Using weakly entangled states

$|\Psi(\theta)>$

ALICE



BOB

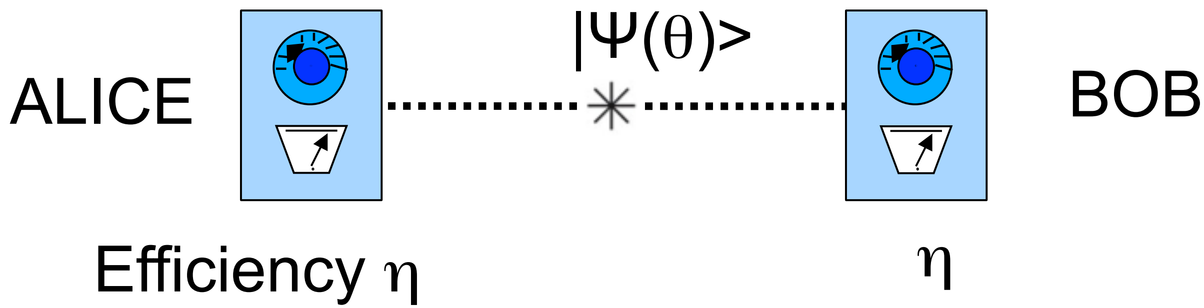
Efficiency η

η

$$|\Psi(\theta)> = \cos\theta\ |0,0> + \sin\theta\ |1,1>$$

Eberhard PRA 1993

Using weakly entangled states



$$|\Psi(\theta)\rangle = \cos\theta\,|0,0\rangle + \sin\theta\,|1,1\rangle$$

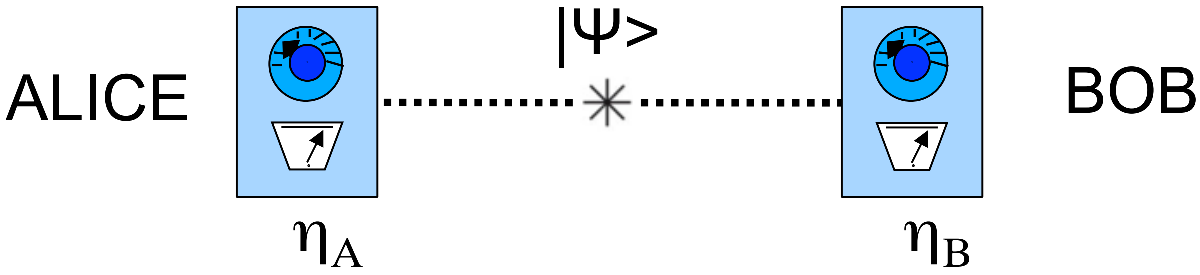
$\theta = \pi/4 \;\rightarrow\; \eta = 82.8\%$

$\theta \rightarrow 0 \;\rightarrow\; \eta \rightarrow 66.7\%$

→ More nonlocality with less entanglement

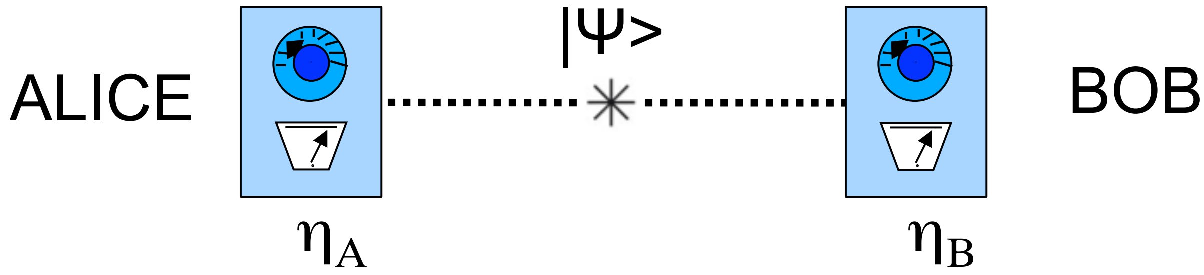
Eberhard PRA 1993

More sophisticated Bell tests



Parameters:
- |Ψ> of dimension d x d
- M measurement settings
- K outcomes

More sophisticated Bell tests



Parameters:
- $|\Psi\rangle$ of dimension d x d
- M measurement settings
- K outcomes

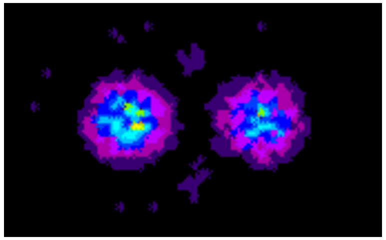
Massar 2005
$\eta \rightarrow 0$ for $d \rightarrow \infty$

Vertesi, Pironio, NB 2010
1. $\eta_A=1/M$, $\eta_B=1$ with d=M, K=2
2. $\eta_A=\eta_B=0.61$ with d=M=4, K=2

Experiments / detection loophole

Atoms

Unit efficiency → No detection loophole



2001 NIST, Rowe et al. Nature

2013 Hoffman et al. Science
Separation of **20m**

Experiments / detection loophole

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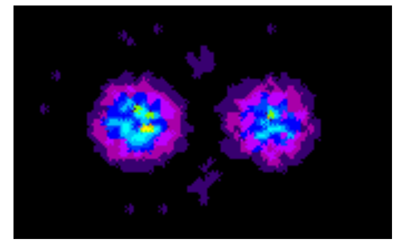

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Superconducting qubits

2009 Ansmann et al. Nature

Experiments / detection loophole

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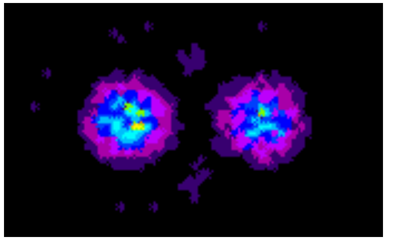


2001 NIST, Rowe et al. Nature

2013 Hoffman et al. Science
Separation of **20m**

Superconducting qubits

2009 Ansmann et al. Nature

Photons

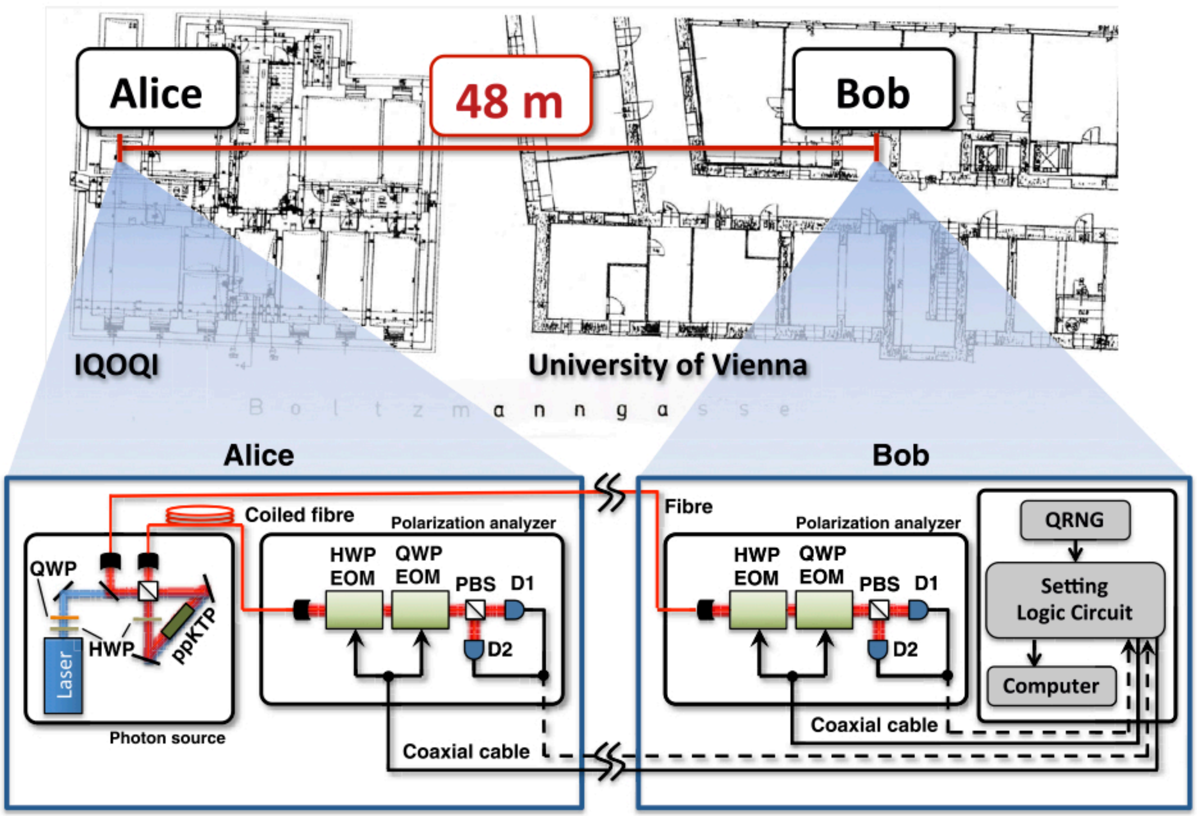
2013 Illinois: Christensen et al. PRL (efficiency 75%)
Vienna: Giustina et al. Nature

ALL EXPERIMENTS SO FAR CONFIRMED Q NONLOCALITY

BUT...

NO EXPERIMENT COULD CLOSE BOTH LOOPHOLES SIMULTANEOUSLY

Loophole-free EPR steering

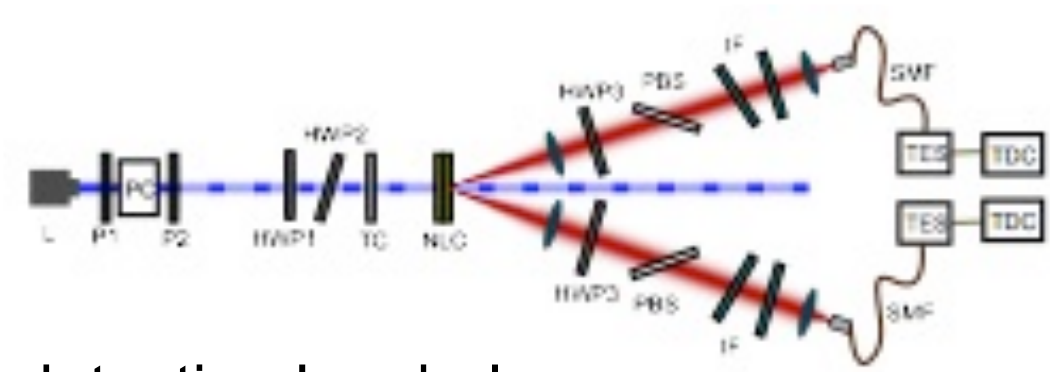


Total efficiency A ~ 38% Steering ineq violated by > 20 σ

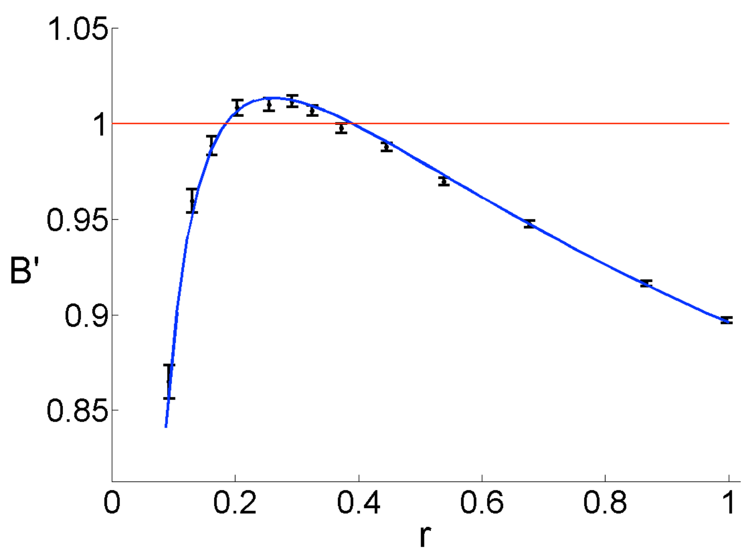
Wittmann et al. NJP 2012

Towards a loophole-free Bell test

Photons: Illinois experiment



Closes detection loophole
TES (superconducting) detectors: efficiency ~ 75%

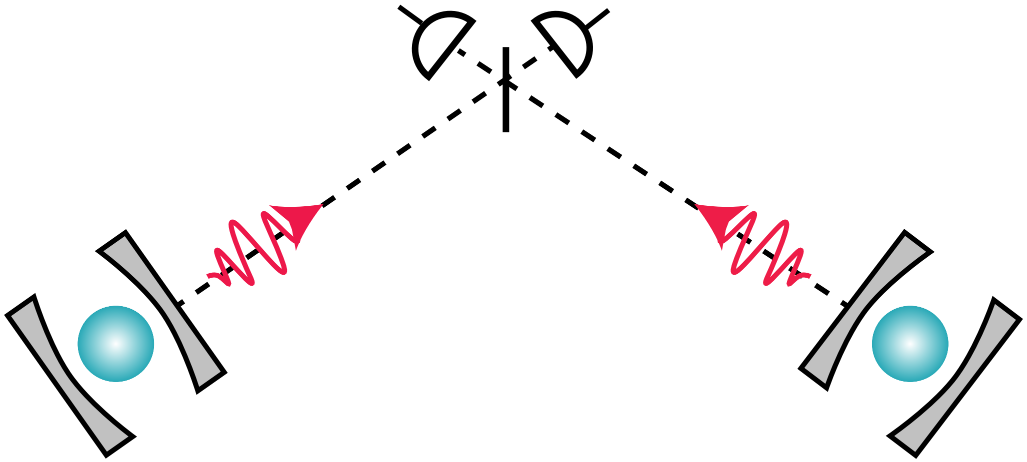


Partially entangled state

$|\psi\rangle$ r | H,H> + | V,V>

Christensen et al. PRL 2013

Atom-photon entanglement



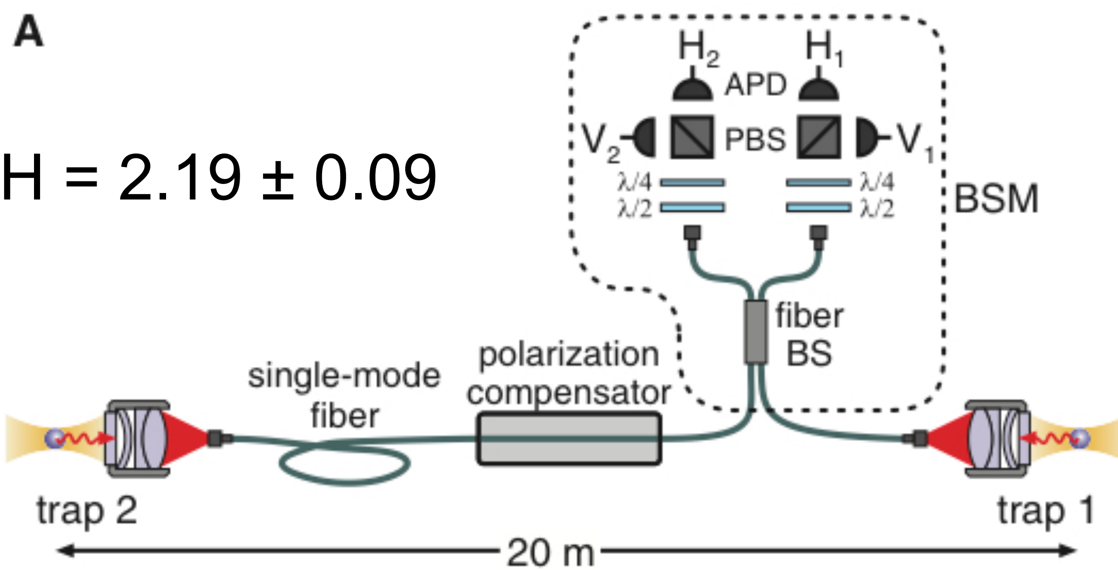
'Event ready' atom-atom entanglement

Simon & Irvine PRL 2003

Atomic Bell test

Bell violation with 2 atoms separated by 20 meters
Munich: Hofmann et al. Science 2012

CHSH = 2.19 ± 0.09



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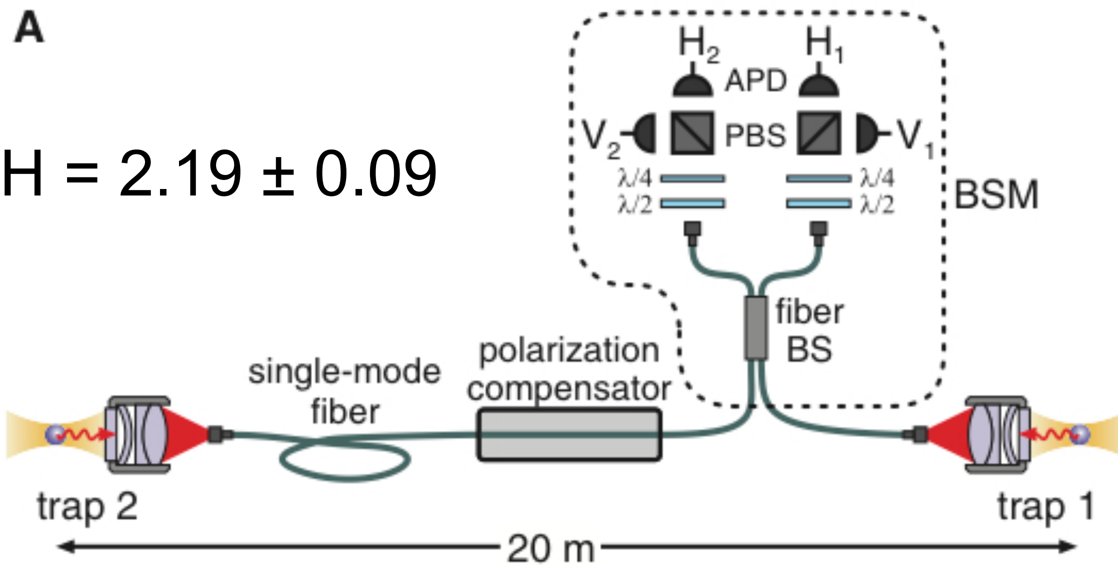


NV centers See talk R. Hanson

Continuous variables

Interest: homodyne measurements have high efficiency

Proposed in 1988 by Grangier et al.

Homodyne measurements

Garcia-Patron et al. PRL06, Nha & Carmichael PRL07

Fatal Post-selection

Babichev et al. PRL04

Homodyne & photodetection (particle and wave)

Cavalcanti et al. PRA11

Continuous variables

Interest: homodyne measurements have high efficiency

Proposed in 1988 by Grangier et al.

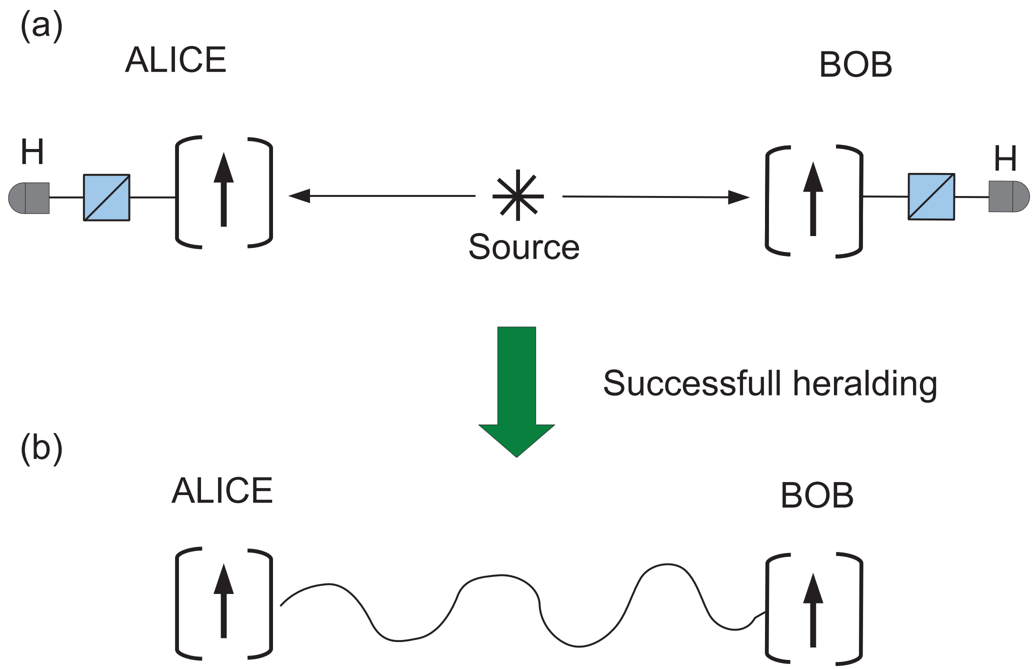
Homodyne measurements
Garcia-Patron et al. PRL06, Nha & Carmichael PRL07

Fatal Post-selection
Babichev et al. PRL04

Homodyne & photodetection (particle and wave)
Cavalcanti et al. PRA11

→ Progress, but still challenging

Spin-photon interactions



Heralded mapping of photonic entanglement to spins

Relevant for atoms, NV centres, Q dots

Sangouard et al. NJP 2013
Brunner et al. NJP 2013

Loopholes in device-independent protocols

Device-independent protocols

GOAL: Achieve information-theoretic task without placing assumptions on the detailed functioning of the devices used in the protocol

Device-independent protocols

GOAL: Achieve information-theoretic task without placing assumptions on the detailed functioning of the devices used in the protocol

Bell inequality violation

↓

Local outcomes A and B are random and uncorrelated from Eve

Device-independent protocols

1. Locality loophole Not so crucial…

→ Alice and Bob must shield their labs anyway

2. Detection loophole **Important !**

Fake Bell violation reported experimentally
Gerhardt et al. PRL 2011, Pomarico et al. NJP 2011

Implementations

DI randomness certification

Proof-of-principle experiments

- Atoms: 42 bits in month Pironio et al. Nature 2010
- Photons: Christensen et al. PRL 2013

DI QKD

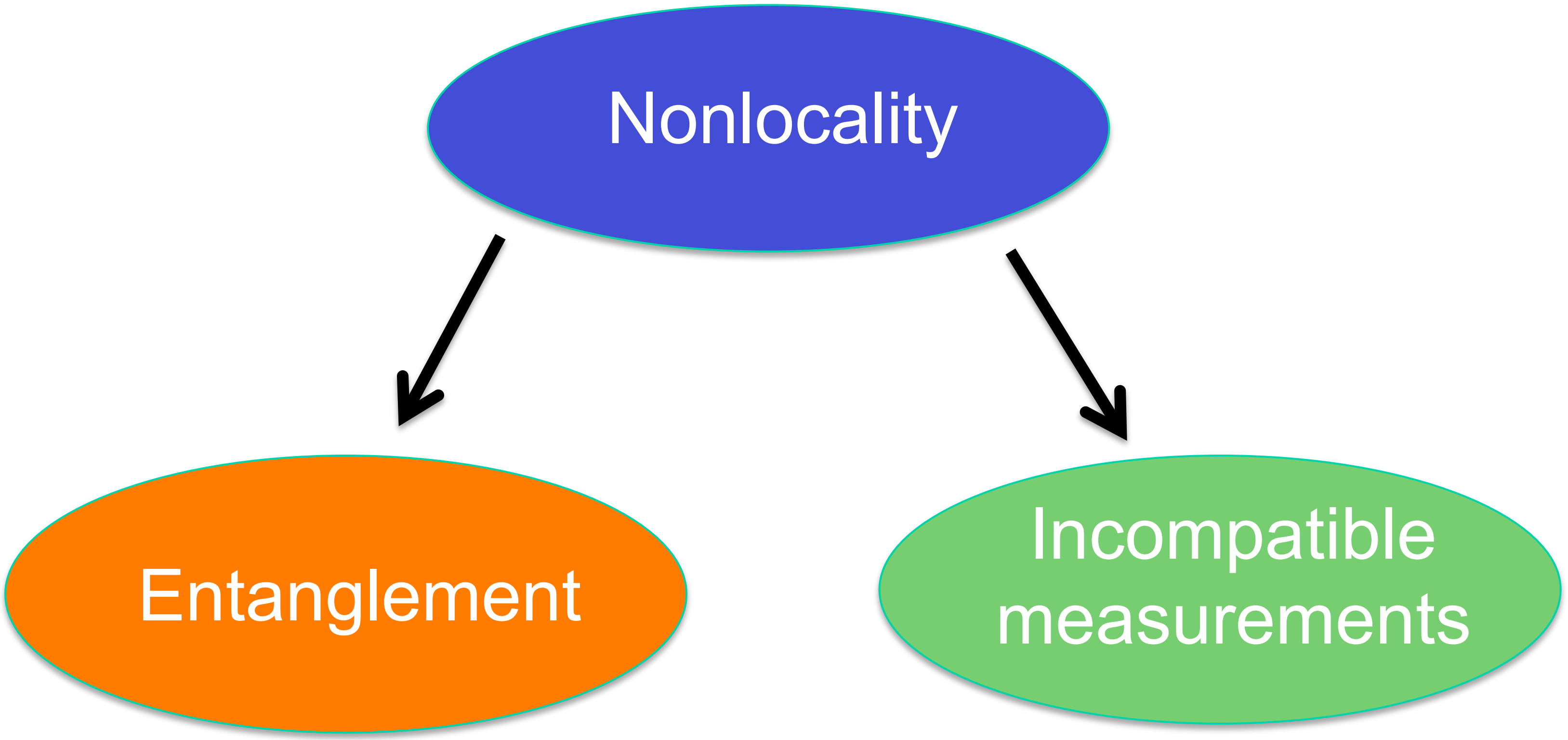
Still challenging

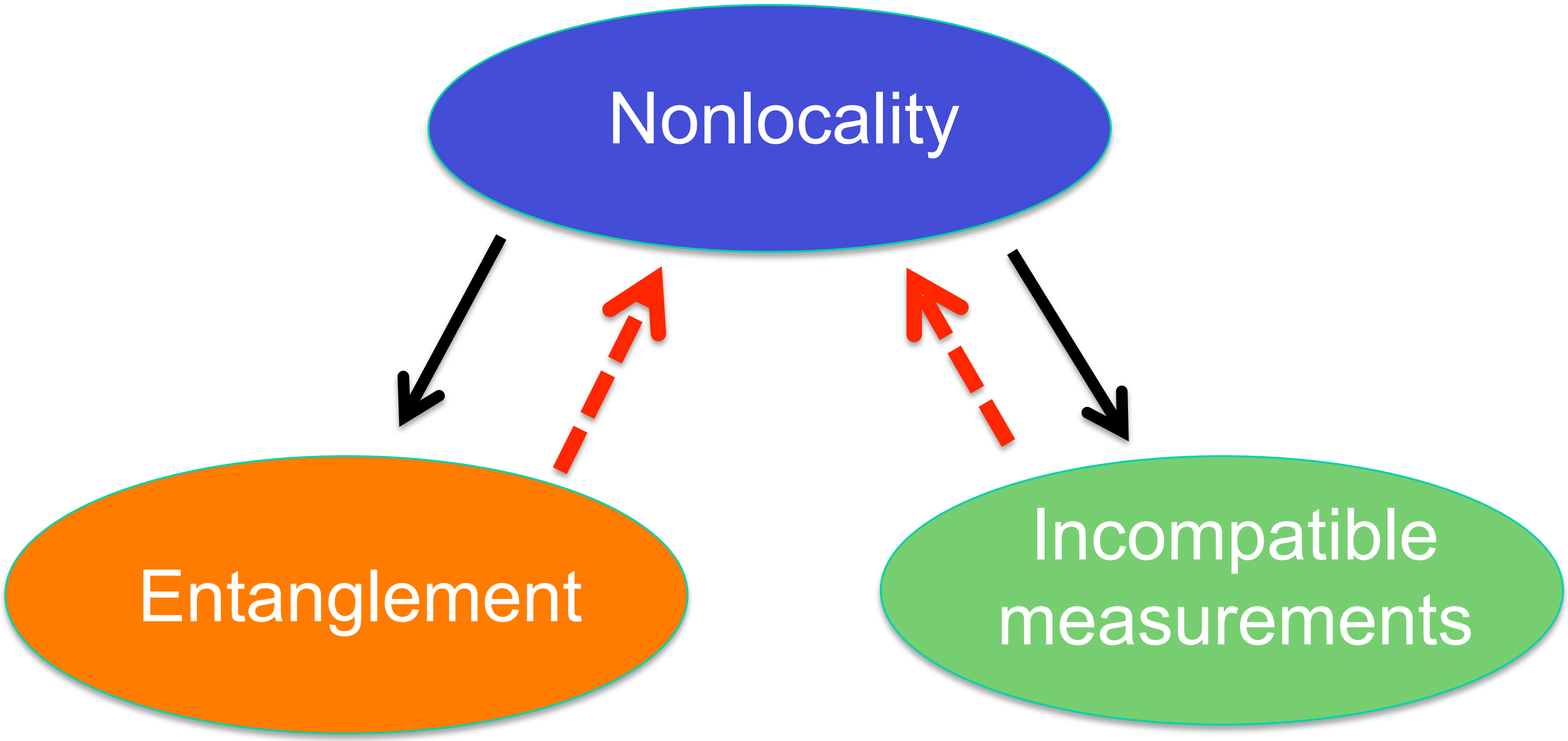
END

References

Brunner, Cavalcanti, Pironio, Scarani, Wehner, RMP 2014

Larsson J Phys A to appear


Nonlocality
Entanglement
Incompatible measurements


Nonlocality
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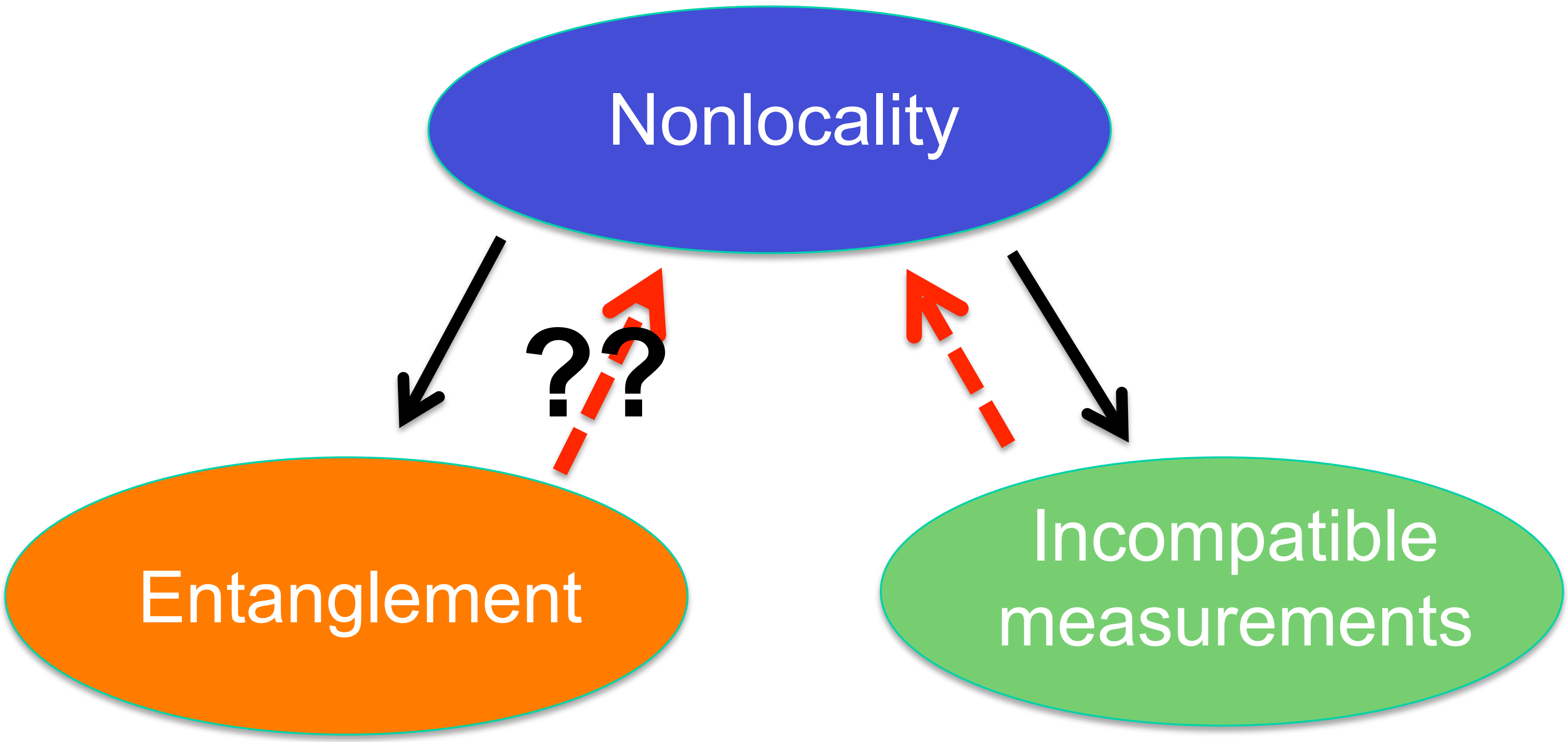
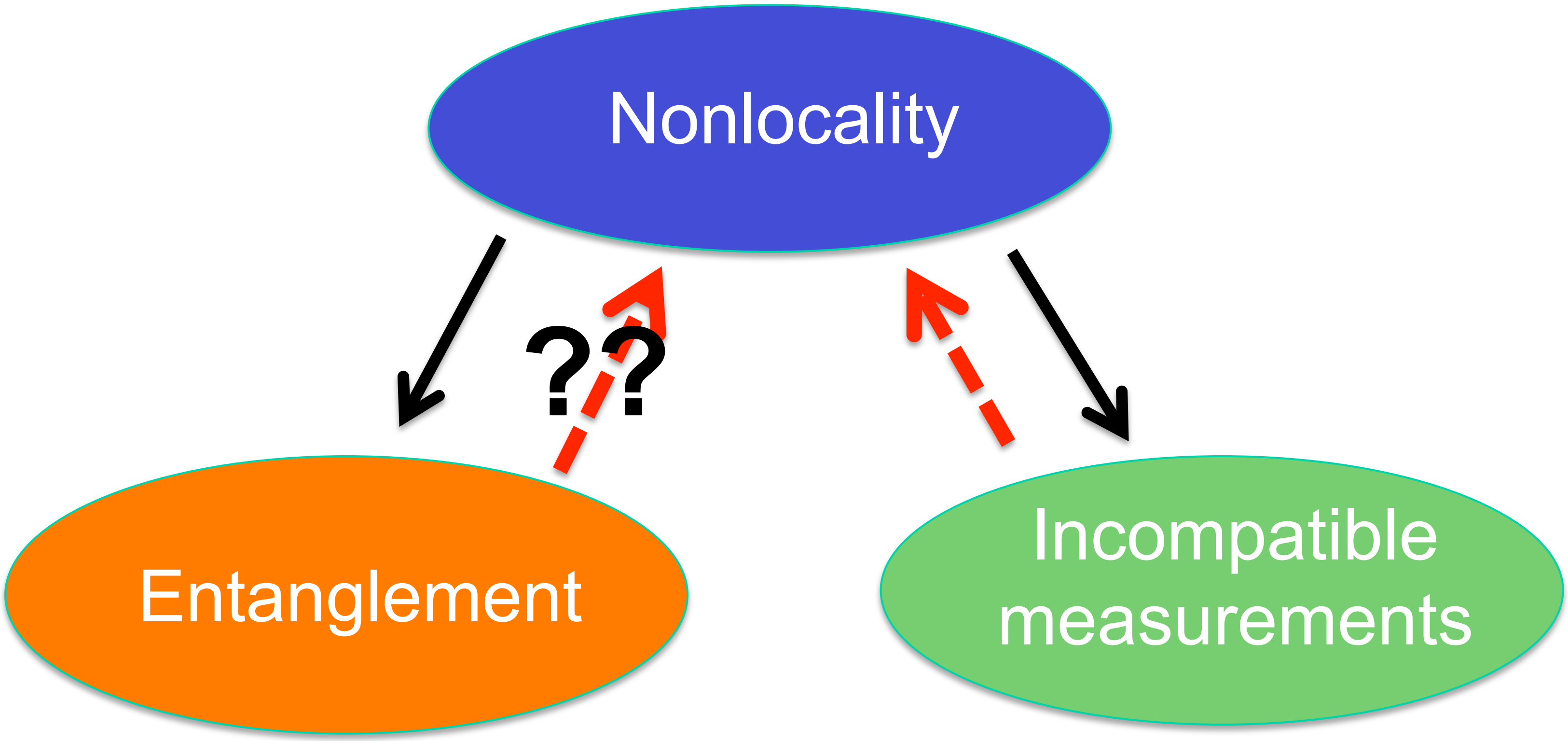


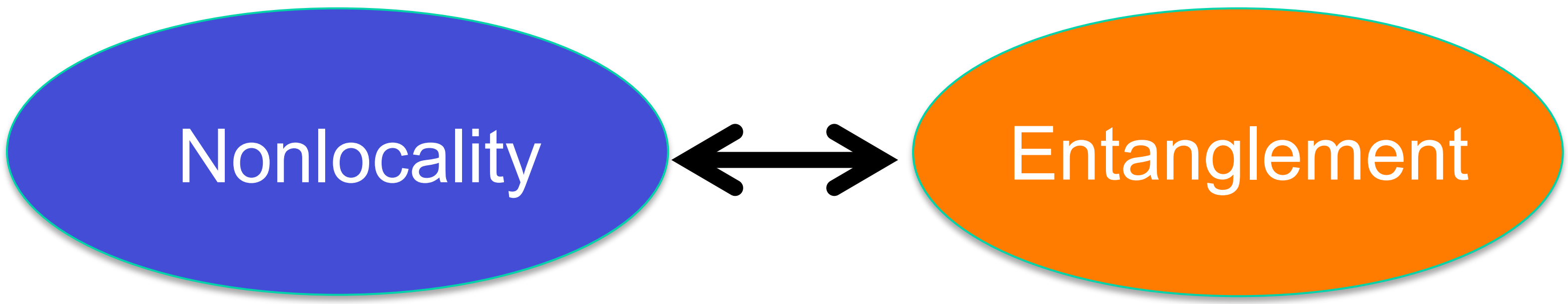
Q1 Do all entangled states violate a Bell inequality?



Q1 Do all entangled states violate a Bell inequality?

Q2 Do all incompatible measurements violate a Bell inequality?

Pure states



All pure entangled states violate a Bell inequality

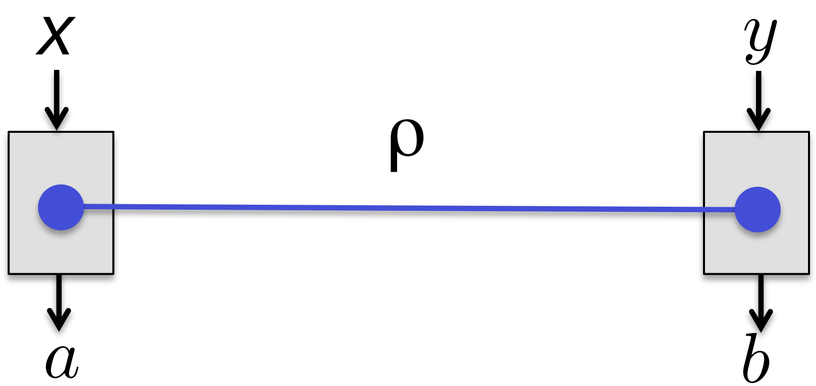
Gisin 91, Popescu-Rohrlich 92

Mixed states…

… complicated!

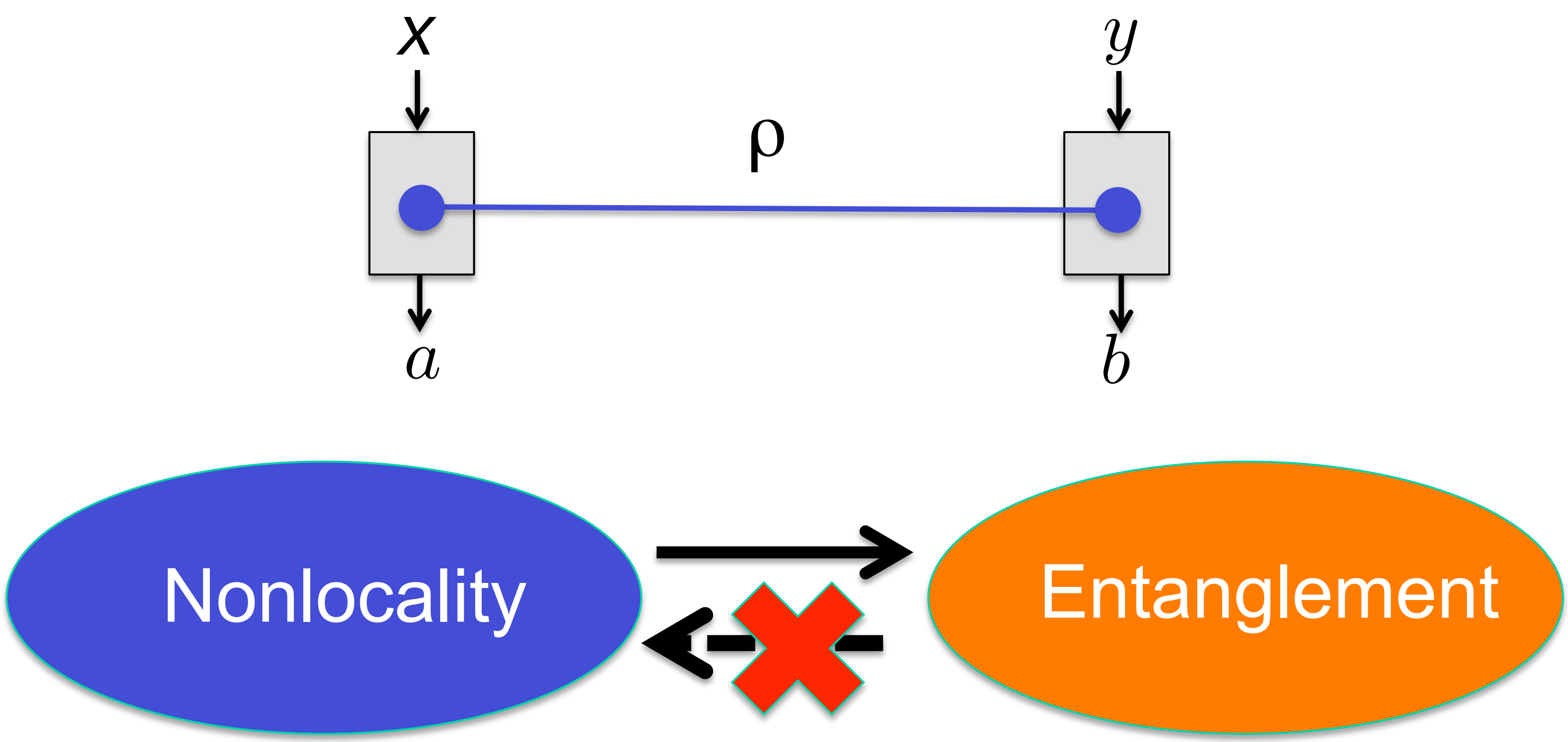
Mixed states…

Scenario 1: Non-sequential measurements



Mixed states…

Scenario 1: Non-sequential measurements

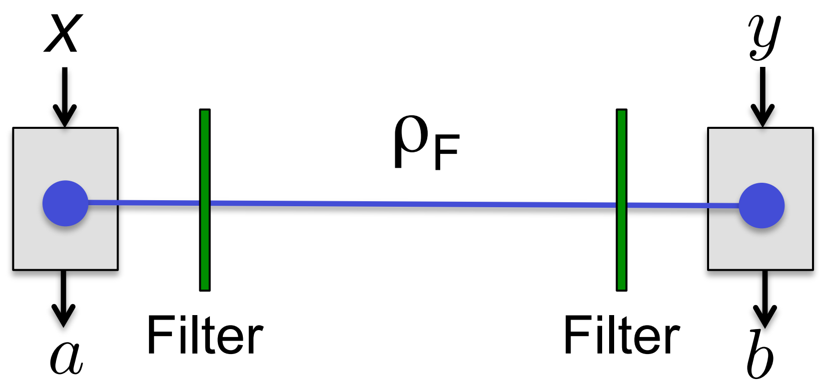


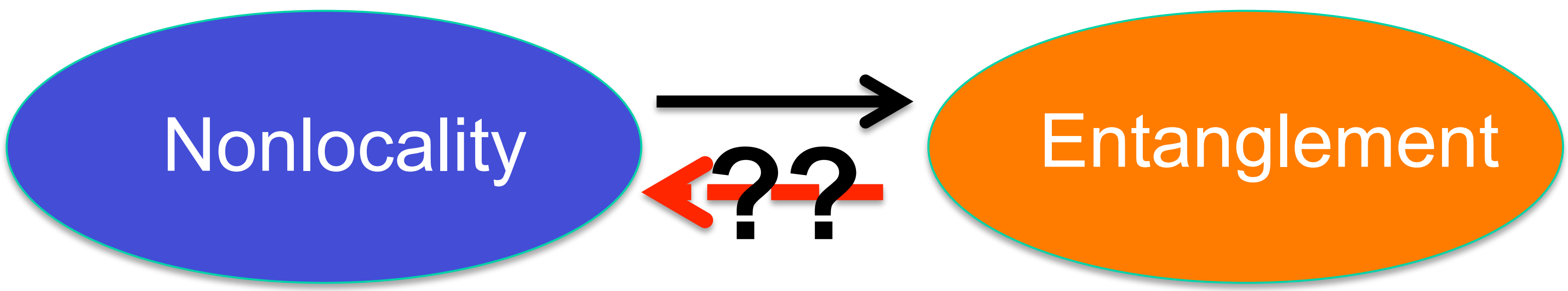
Werner 89, Barrett 2002

Mixed states…

Scenario 2: Sequential measurements

→ **Hidden nonlocality**



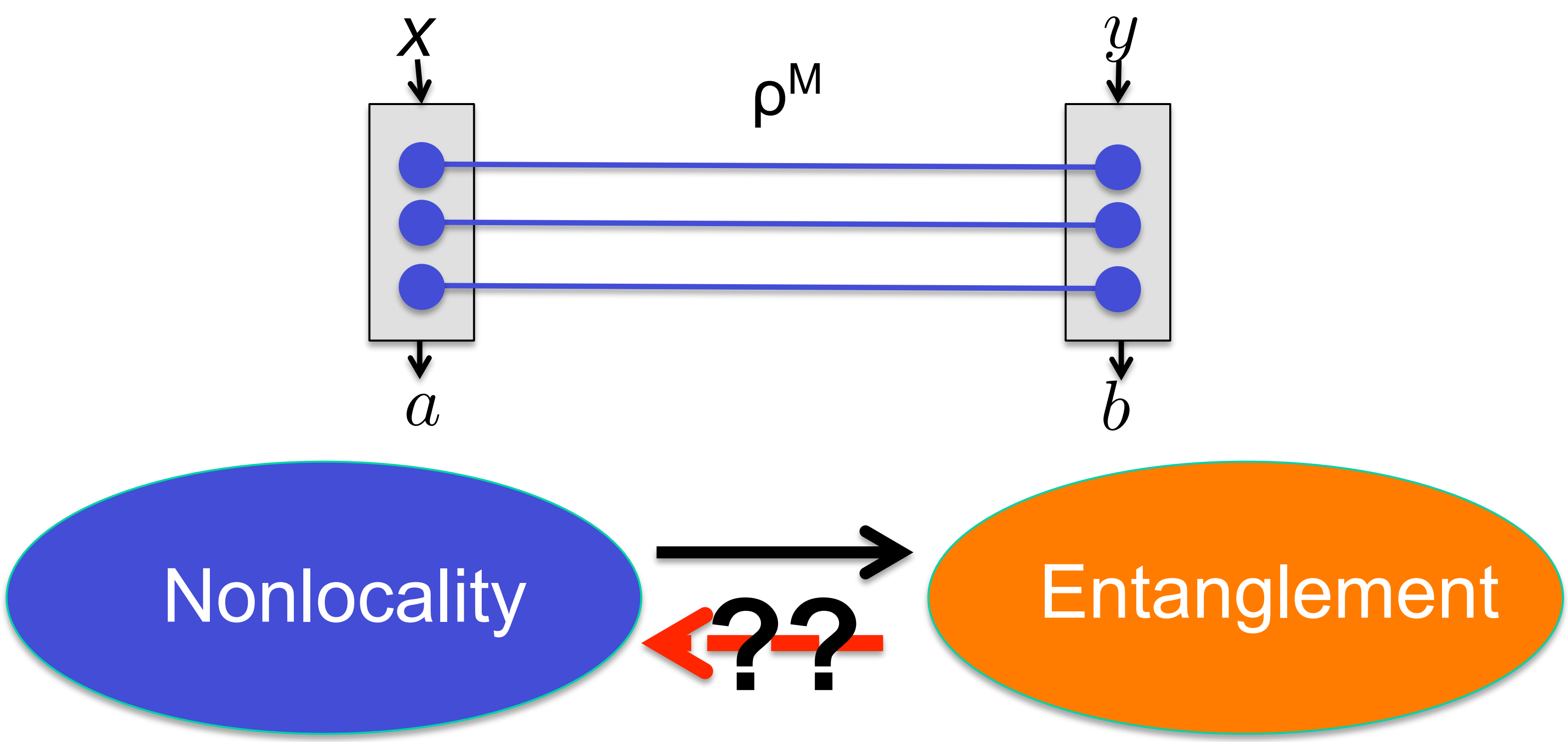


Popescu 95, Hirsch et al. 2013

Mixed states…

Scenario 3: Many copies, joint measurements

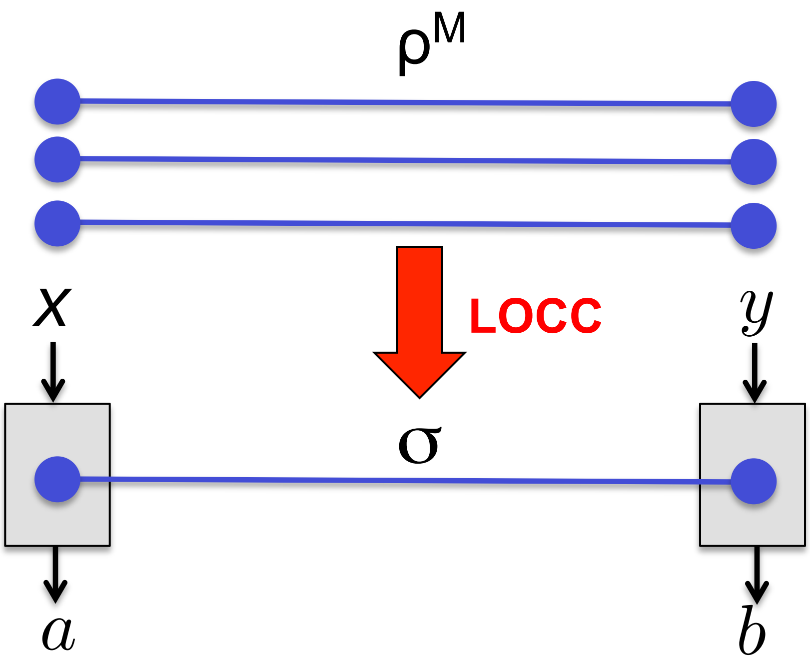
→ **Activation of nonlocality**



Palazuelos 2012

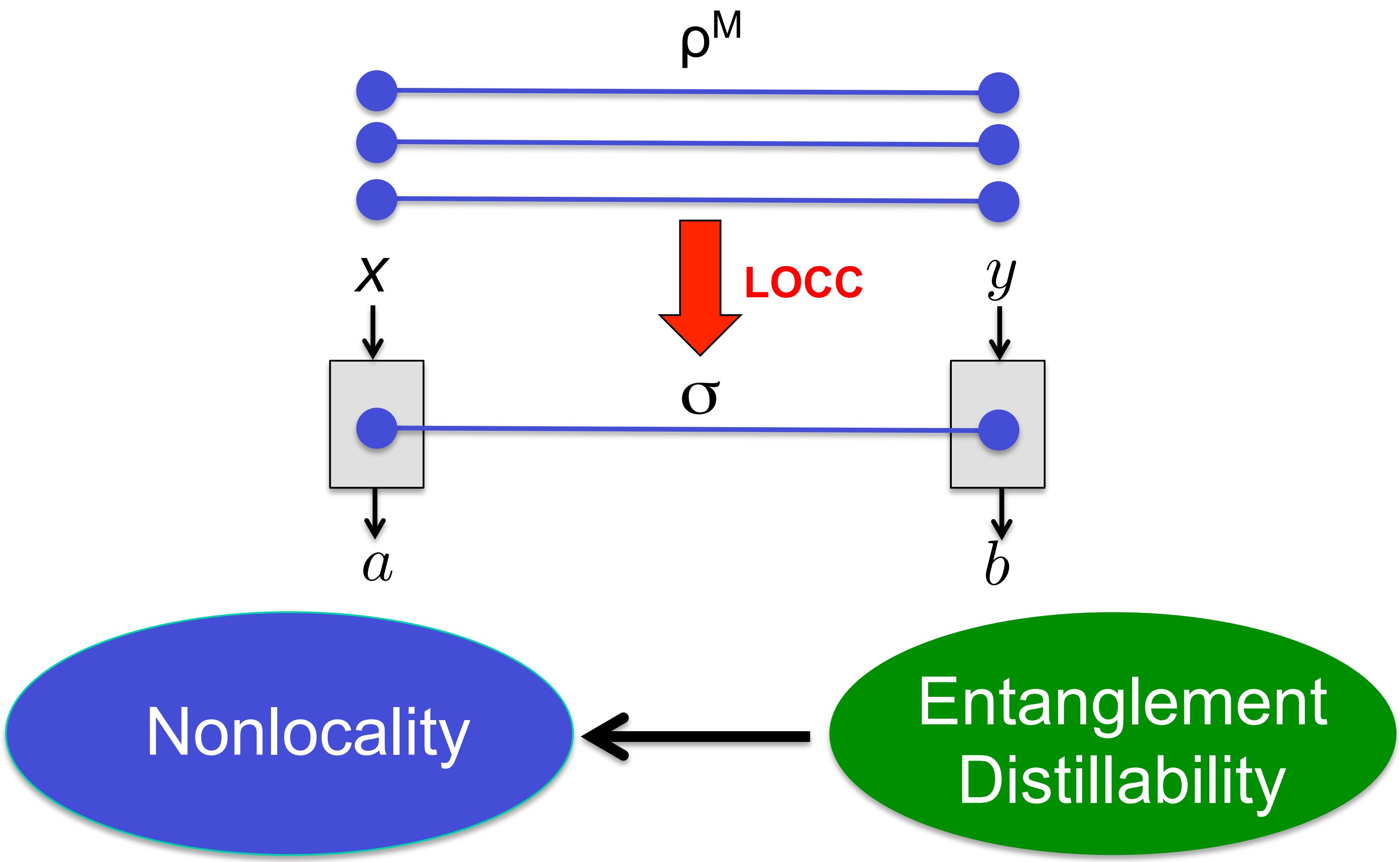
Mixed states…

Scenario 4: Many copies, LOCC before Bell test



Mixed states…

Scenario 4: Many copies, LOCC before Bell test



Peres 96, 99

What about bound entanglement?

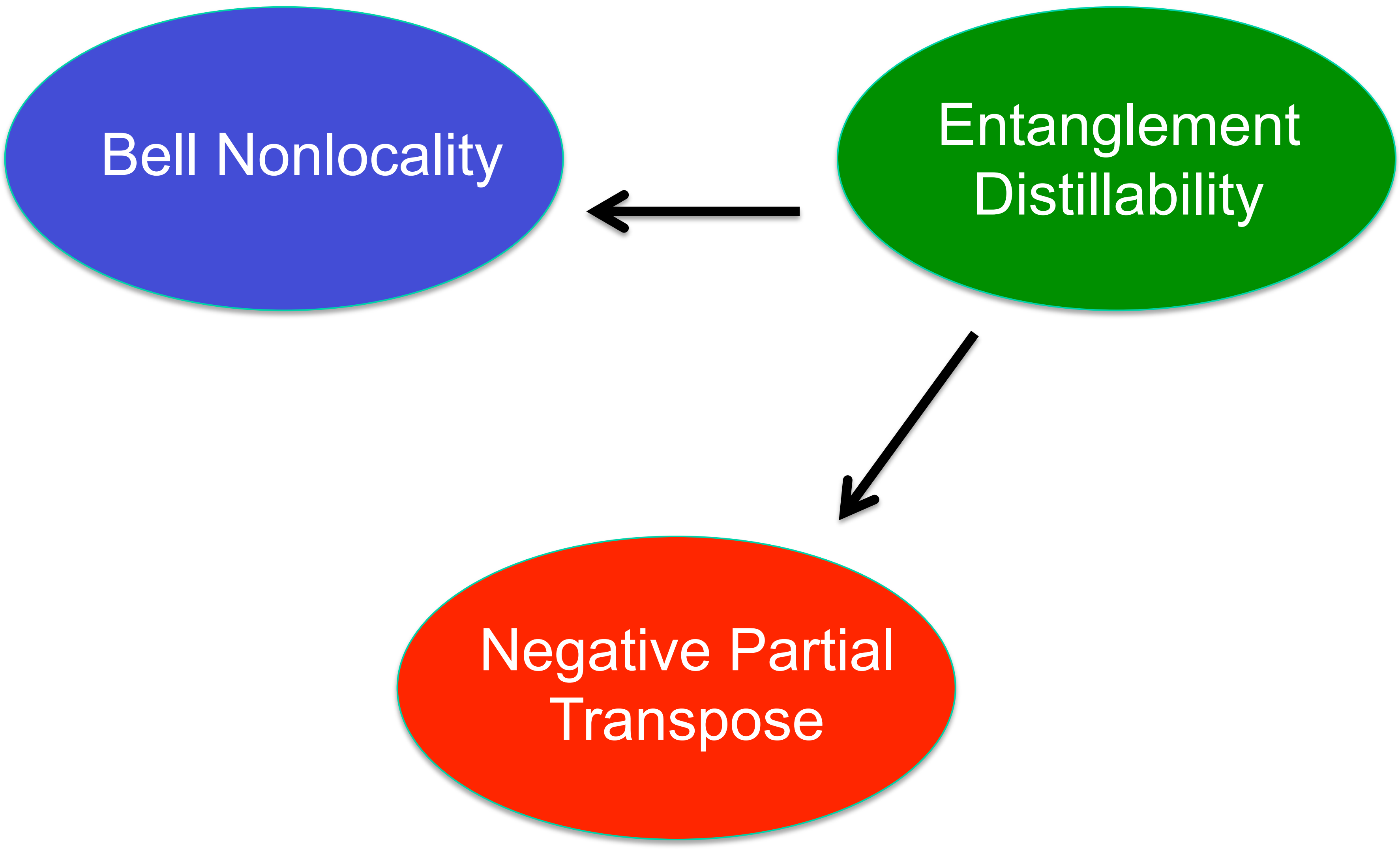
Peres conjecture (1999):

Bound entanglement cannot lead to Bell inequality violation

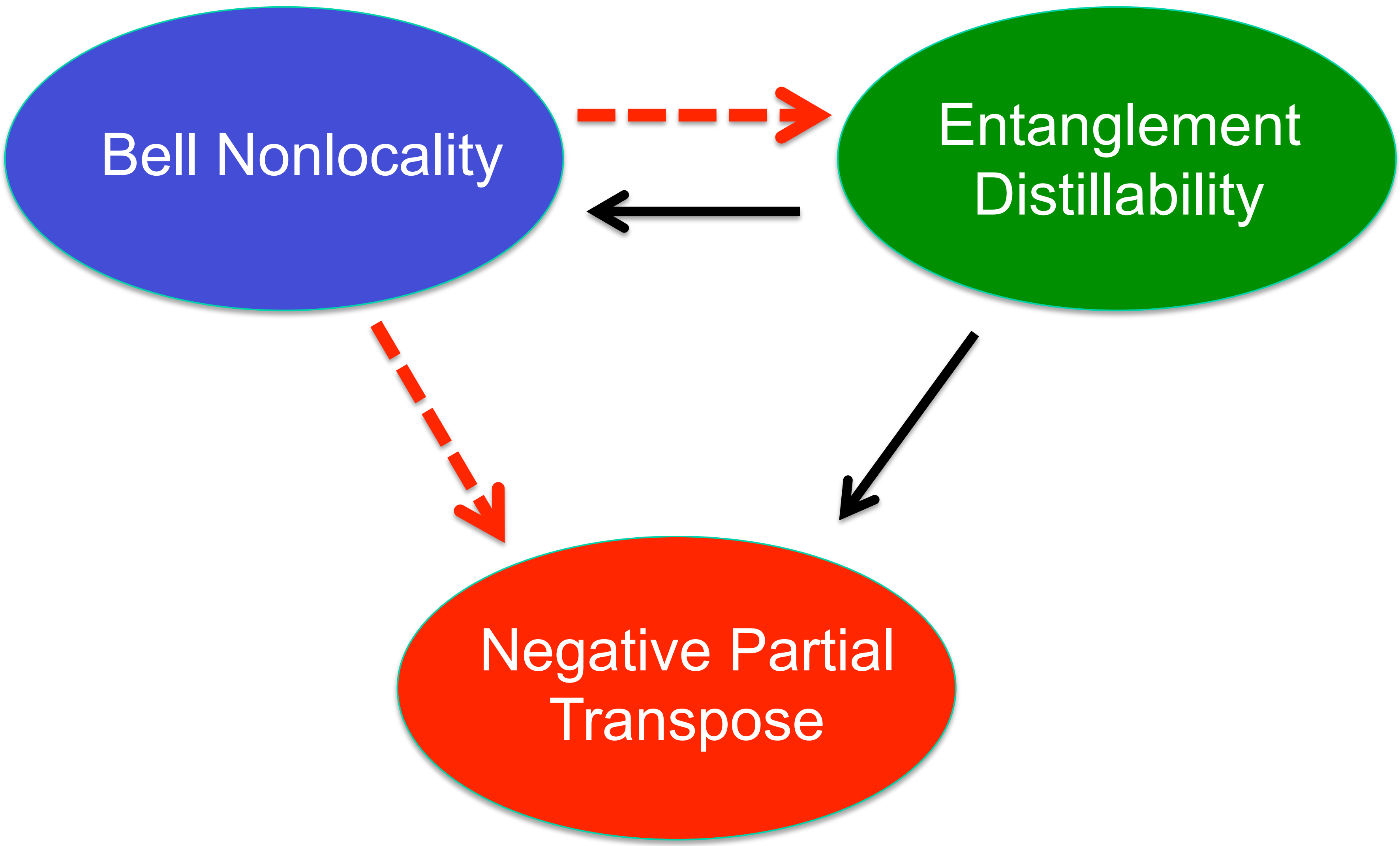
Intuition:

weakest form of entanglement cannot lead
to strongest correlations

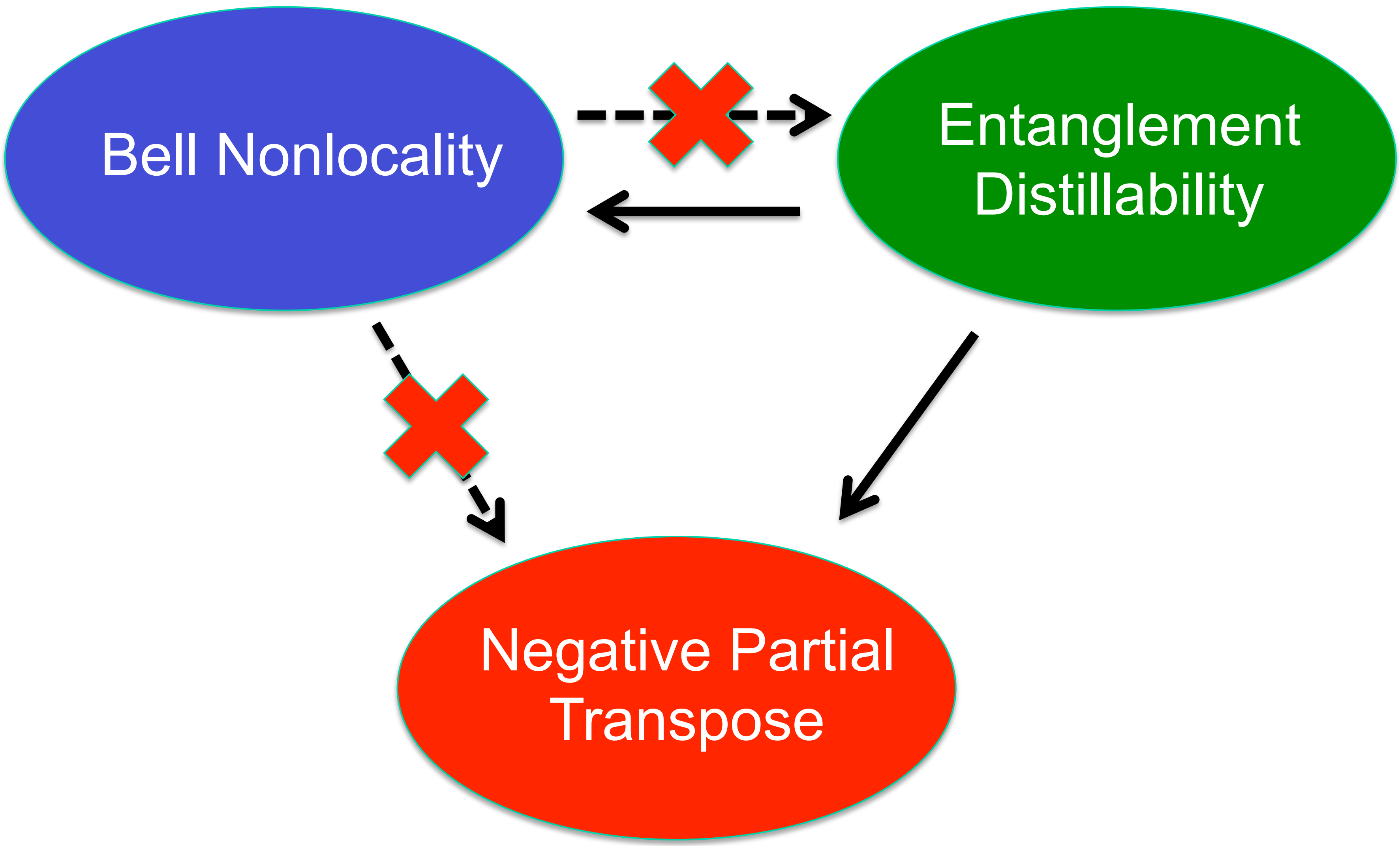
Peres 1999


Bell Nonlocality
Entanglement Distillability
Negative Partial Transpose

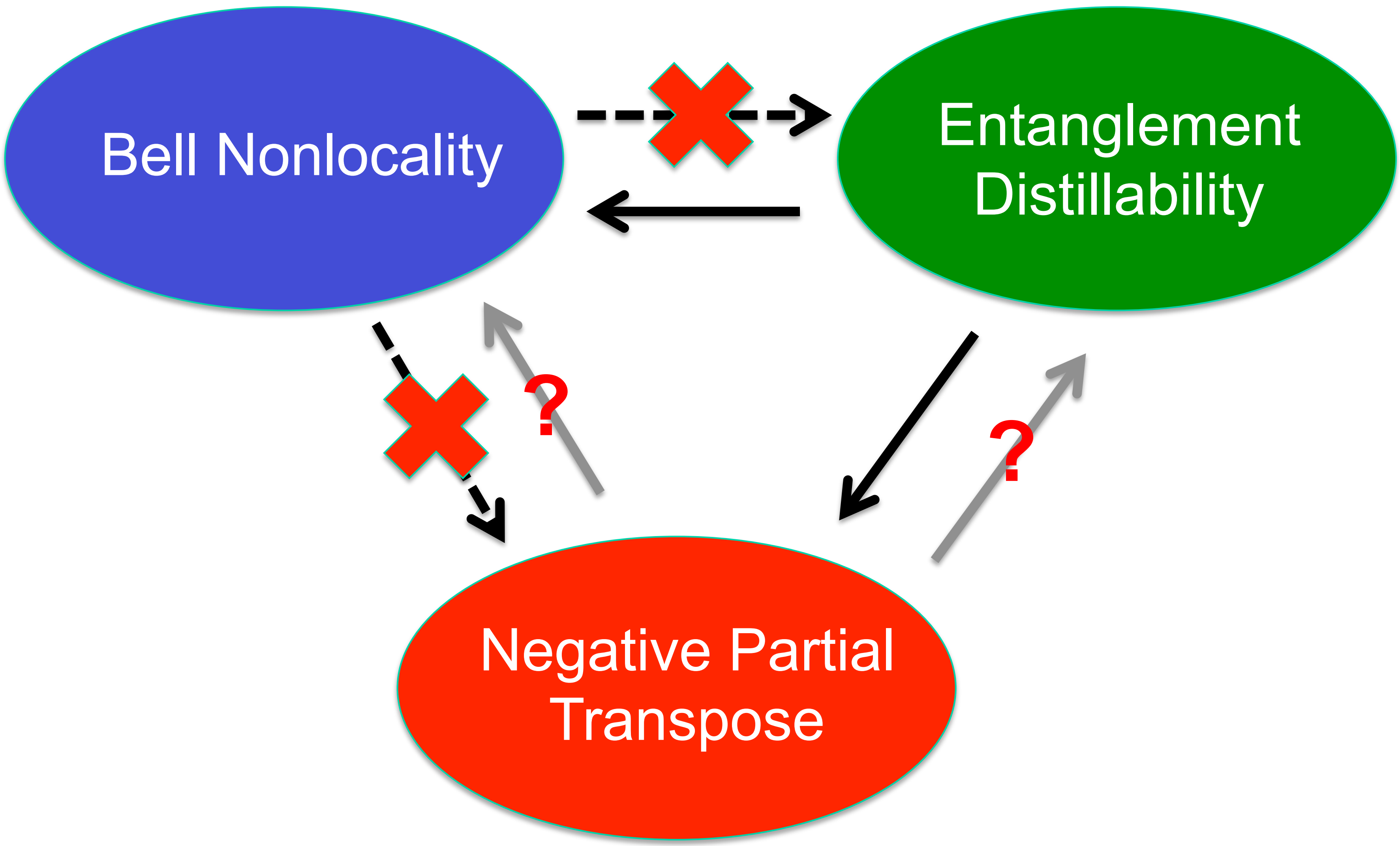
Peres conjecture (1999)…



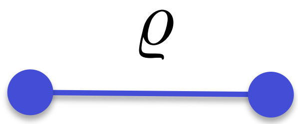
… is false


Bell Nonlocality
Entanglement Distillability
Negative Partial Transpose

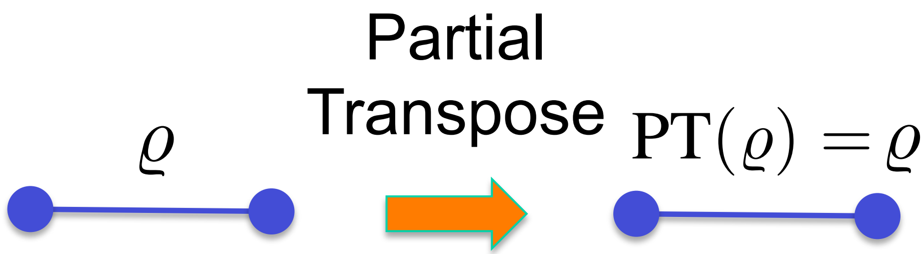
NPT bound entanglement?



Building a counter-example



Building a counter-example



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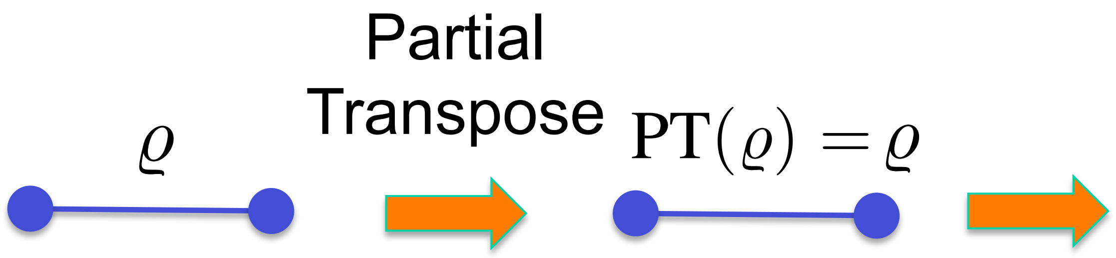
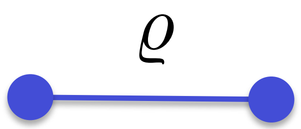


(1) ϱ is PPT
→ undistillable

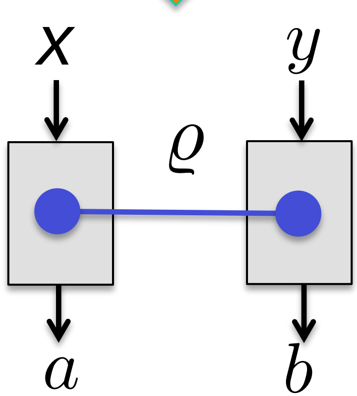
Building a counter-example



Partial Transpose

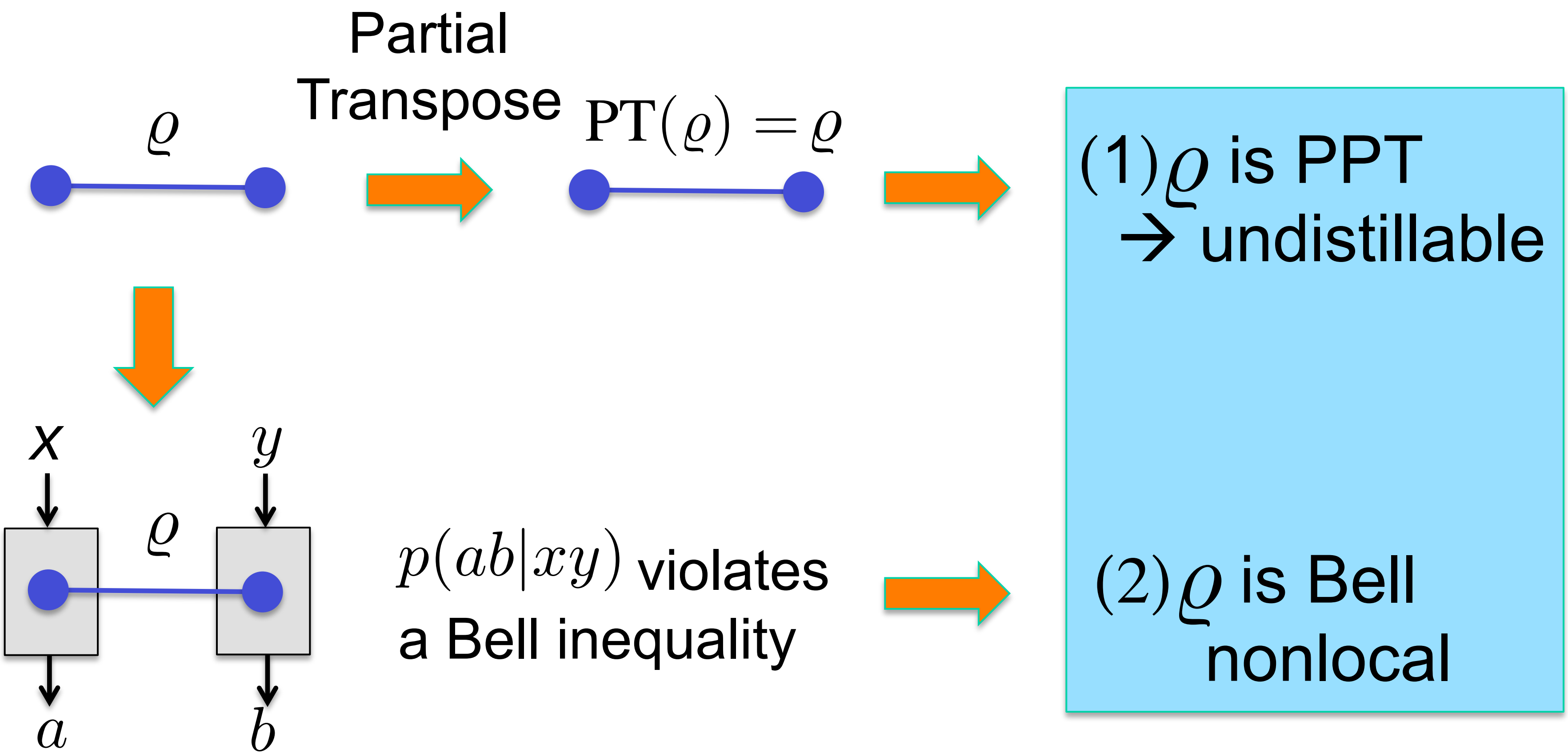
$\mathrm{PT}(\varrho) = \varrho$

(1) ϱ is PPT
→ undistillable

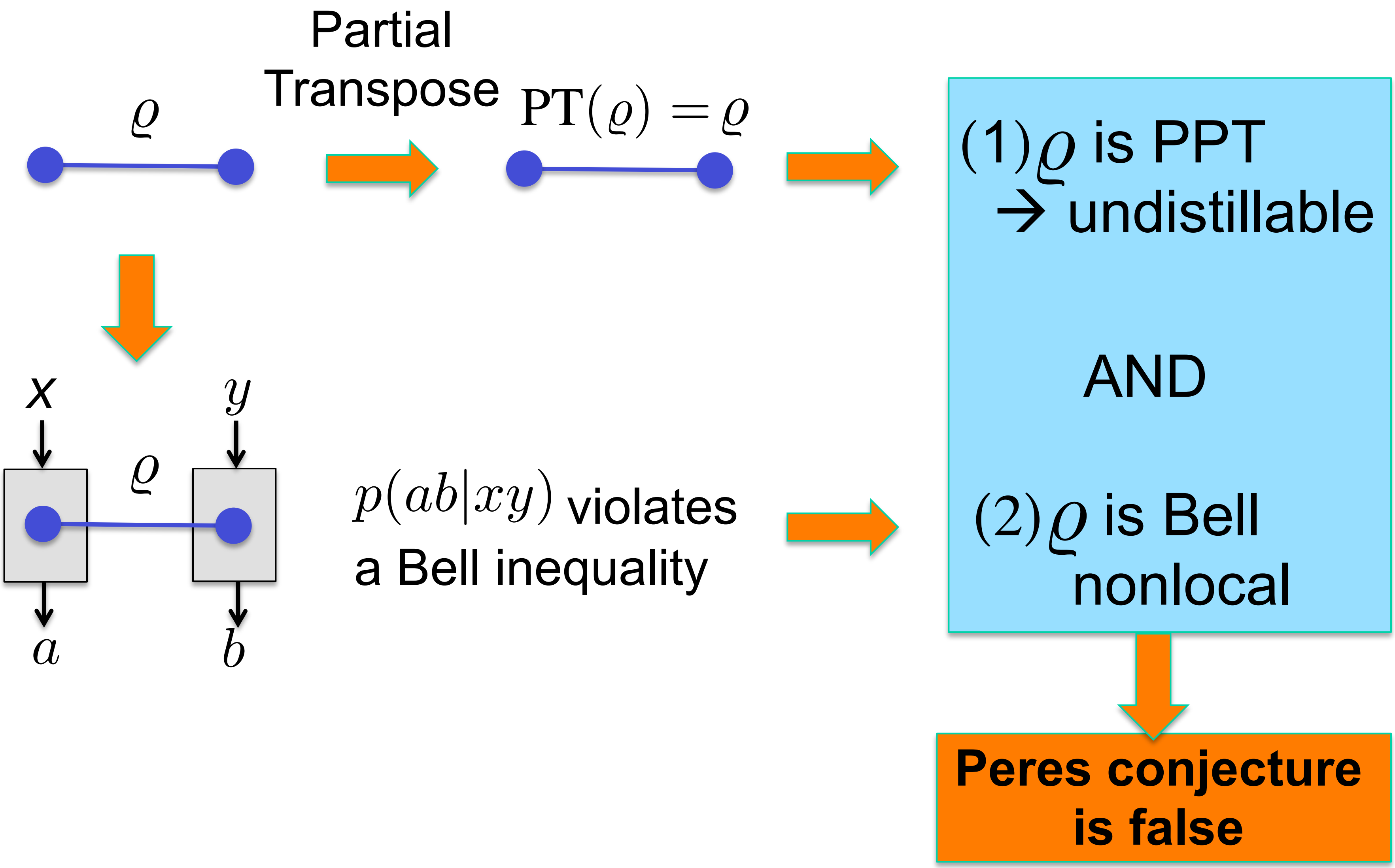


$p(ab|xy)$ violates a Bell inequality

Building a counter-example



Building a counter-example



Details

State: 3 x 3 (Moroder et al 2014)

$$\varrho = \sum_{i=1}^{4} \lambda_i |\psi_i\rangle\langle\psi_i|.$$

$$\lambda = \left(\tfrac{3257}{6884}, \tfrac{450}{1721}, \tfrac{450}{1721}, \tfrac{27}{6884}\right)$$

$$a = \sqrt{\tfrac{131}{2}}$$

$$|\psi_1\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

$$|\psi_2\rangle = \frac{a}{12}\left(|01\rangle + |10\rangle\right) + \frac{1}{60}|02\rangle - \frac{3}{10}|21\rangle$$

$$|\psi_3\rangle = \frac{a}{12}\left(|00\rangle - |11\rangle\right) + \frac{1}{60}|12\rangle + \frac{3}{10}|20\rangle$$

$$|\psi_4\rangle = \frac{1}{\sqrt{3}}\left(-|01\rangle + |10\rangle + |22\rangle\right),$$

Details

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$$\mathrm{PT}(\varrho) = (\mathbb{1} \otimes T_B)(\varrho) = \varrho$$

→ ϱ is PPT → undistillable

Details

Bell inequality (Pironio 2014)**:**

Alice: 3 binary meas
Bob: 1 ternary meas, 1 binary meas

$$I = -p_A(0|2) - 2p_B(0|1) - p(01|00) - p(00|10) + p(00|20) + p(01|20) + p(00|01) + p(00|11) + p(00|21) \leq 0,$$

Details

Bell inequality (Pironio 2014)**:**

Alice: 3 binary meas
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$$\begin{aligned} I \;=\; & -p_A(0|2) - 2p_B(0|1) - p(01|00) - p(00|10) + p(00|20) \\ & + p(01|20) + p(00|01) + p(00|11) + p(00|21) \leq 0, \end{aligned}$$

ϱ + well chosen measurements

$$I_\varrho = \frac{-3386 + 18\sqrt{42} - 5\sqrt{131} + 45\sqrt{5502}}{43025} \simeq 2.63144 \times 10^{-4}$$

ϱ is nonlocal

SDP methods

SDP technique to find Bell inequality violation with PPT state

$$I_{PPT} = 2.6526 \times 10^{-4}$$

Upper bound (Moroder et al. 2013)

$$I_{PPT}^{max} < 4.8012 \times 10^{-4}$$

Applications

1. Device-independent randomness certification
   (Pironio et al. Nature 2010, Colbeck PhD thesis 2007)

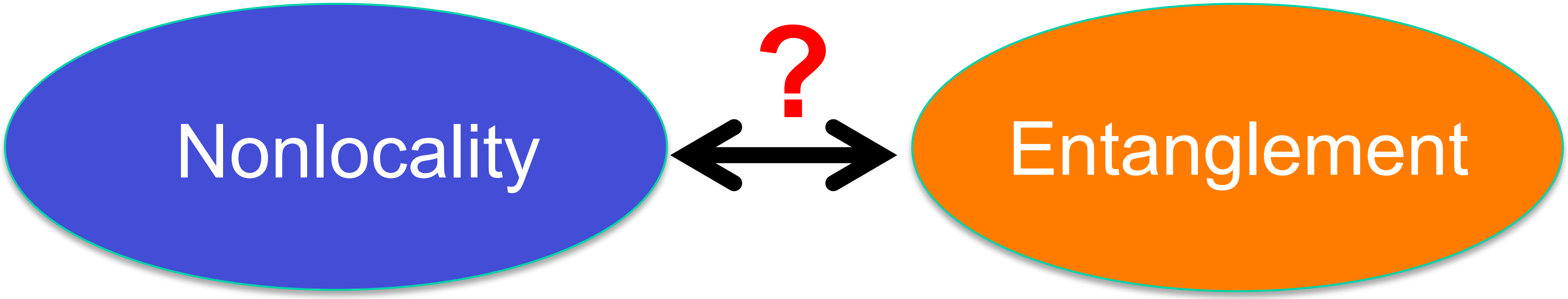
Quantum nonlocality → genuine quantum randomness

Bell violation I_{PPT}	$H_{min}\ (y=0)$
2.6314×10^{-4}	4.2320×10^{-4}

2. Communication complexity
   (Zukowski et al. 2004, Buhrman et al. 2010)

Open questions

1. Do all BE states violate a Bell inequality?



2. Large (unbounded) Bell violations with BE state?

3. Device-independent QKD with BE state?

END

References

Brunner, Cavalcanti, Pironio, Scarani, Wehner, RMP 2014

Larsson J Phys A to appear