Continuous Variable Quantum Key Distribution: Finite-Key Analysis of Composable Security against Coherent Attacks

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Joint work with
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Security of a QKD Protocol

Constraints:
- Information theoretic
- Experimental

Experimental implementation

Security Analysis

-Key length
-Certification of security

Secure Key

Minimizing the assumption and maximizing the key length!
Security of a QKD Protocol

Constraints:

- Information theoretic
  - Asymptotic key rate vs. finite uses of QM channel (finite-key effects)
  - Notion of security: composable?
  - Limitation on attacks: collective (tensor product) or coherent (general)?
  - ...

- Experimental / Implementation
  - Model of the measurement devices
  - Model of the quantum source
  - ...

FF, Continuous Variable QKD: Finite-Key Analysis against Coherent Attacks, Singapore, 14.09.2012
Contribution: Security analysis for continuous variable (CV) protocol based on the distribution of two-mode squeezed states (EPR states) measured via homodyne detection.
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What’s New: Computation of key length secure against coherent attacks for achievable experimental parameters.

Discrete Variables vs. Continuous Variables

Implementation

- Encoding in finite-dimensional systems (e.g., polarization of photon)

- Encoding in infinite-dimensional systems (bosonic modes) [1]
  - Gaussian States
  - Quadratures of EM-field: Homodyne or Heterodyne detection

Advantage:
- Compatible with standard telecom technology
- High repetition rates for homodyne
- Efficient state preparation

Security Analysis for CV QKD Protocols

**Challenge:** infinite dimensions

**Finite-Key Analysis:**

**Lifting proofs from collective to coherent (general) attacks:**
- Exponential de Finetti [Renner & Cirac, PRL 102, 110504 (2009)]
  - **Problem:** Bad bounds, feasible only in the asymptotic limit
- Post-selection technique,
  - **Recent:** Leverrier et al., arXiv:1208.4920 (Talk on Monday)
Security Analysis for CV QKD Protocols

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**Finite-Key Analysis:**

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**Uncertainty Relation (direct):** This Talk!

**Advantage:**
- one-sided device independent
- no tomography
- no additional measurements
Outline

1. Security Definition and Finite-key length formula
2. Experimental Set Up and Protocol
3. Finite-Key Rates
4. (Security Analysis)
General QKD Protocol

Part 1:
1) Distribution of quantum state
2) Measurements
3) Parameter estimation
4) Output: Raw keys $X_A$, $X_B$ or abort
General QKD Protocol

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Part 2:
1) Error correction
2) Privacy amplification
Output: Key $S_A, S_B$
Security Definitions (trace distance)

A protocol which outputs the state

$$\omega_{S_A S_B E}$$

is secure if it is:

- **correct**: $\text{Prob}[S_A \neq S_B] \leq \varepsilon_c$

- **secret**: $p_{\text{pass}} \cdot \|\omega_{S_A E} - \tau_{S_A} \otimes \omega_E\|_1 \leq \varepsilon_s$

where $\tau_{S_A}$ is the uniform distribution over all keys.

*Composable Secure*

1) **Error Correction:**
   Alice and Bob broadcast $\ell_{EC}$ bits to match their strings.

2) **Privacy amplification via two-universal hash functions:**
   ... apply random hash function from two-universal family onto $\ell$ bits

$$f : X_A \rightarrow S_A$$

Key length
Classical Post Processing

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   \[ f : X_A \rightarrow S_A \]

   Secure key of length:

   \[ \ell \approx H_{\min}^\varepsilon (X_A | E)_{\omega} - \ell_{EC} - O(\log \frac{1}{\varepsilon'}) \]

   Smooth min-entropy

M. Berta, FF, V.B. Scholz, arXiv:1107.5460 (infinite-dimensional side-information)
Classical Post Processing

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Smooth min-entropy

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Use parameter estimation to bound min-entropy!
Experimental Set Up

**Source:**
two-mode squeezed state (EPR state)

**Measurements:**
homodyne detection, randomly either amplitude or phase (synchronized via LO)

**Entanglement based!**

Measurements

Correlated outcomes if both measure amplitude or phase:

Source:
two-mode squeezed state

Measurements:
homodyne detection, randomly either amplitude or phase (synchronized via LO)

Entanglement based!
Measurements

Binning of the Outcome Range:

- **Spacing parameter:** $\delta$
- **Cutoff parameter:** $\alpha$

\[
I_1 = (-\infty, -\alpha + \delta]
\]
\[
I_k = (-\alpha + (k - 1)\delta, -\alpha + (k - 2)\delta]
\]
\[
I_{2\alpha/\delta} = (\alpha - \delta, \infty)
\]

Outcome Range:

\[
\mathcal{X} = \{1, 2, \ldots, 2\alpha/\delta\}
\]
Protocol

1. Performing 2N measurements

2. **Sifting**: approx. N data points left \( X_A^{tot}, X_B^{tot} \in \mathcal{X}^N \)

3. **Parameter estimation:**
   - pick random sample of k data points \( Y_A, Y_B \in \mathcal{X}^k \) and check correlation:
   - Hamming distance:
     \[
     d(Y_A, Y_B) = \frac{1}{k} \sum_{i=1}^{k} |Y_A^i - Y_B^i|
     \]

4. **Classical post-processing** on remaining strings \( X_A, X_B \in \mathcal{X}^n \):
Protocol

1. Performing 2N measurements

2. **Sifting**: approx. $N$ data points left $X_A^{\text{tot}}, X_B^{\text{tot}} \in \mathcal{X}^N$

3. **Parameter estimation:**
   
   pick random sample of $k$ data points $Y_A, Y_B \in \mathcal{X}^k$ and check correlation:
   Hamming distance:
   
   $$d(Y_A, Y_B) = \frac{1}{k} \sum_{i=1}^{k} |Y_A^i - Y_B^i|$$

4. **Classical post-processing** on remaining strings $X_A, X_B \in \mathcal{X}^n$:

   A secret key of length
   
   $$\ell = n\left[\log \frac{1}{c(\delta)} - \log \gamma (d(Y_A, Y_B) + \mu)\right] - O(\log \frac{1}{\varepsilon}) - \ell_{EC}$$

   can be extracted
Protocol

1. Performing 2N measurements

2. **Sifting**: approx. N data points left $X_A^{tot}, X_B^{tot} \in \mathcal{X}^N$

3. **Parameter estimation:**
   
   pick random sample of k data points $Y_A, Y_B \in \mathcal{X}^k$ and check correlation:
   
   Hamming distance:
   
   $$d(Y_A, Y_B) = \frac{1}{k} \sum_{i=1}^{k} |Y_A^i - Y_B^i|$$

4. **Classical post-processing** on remaining strings $X_A, X_B \in \mathcal{X}^n$:

A secret key of length

$$\ell = n\left[\log \frac{1}{c(\delta)} - \log \gamma(d(Y_A, Y_B) + \mu)\right] - O\left(\log \frac{1}{\epsilon}\right) - \ell_{EC}$$

can be extracted

Statistical correction

Monotonic function

Complementarity of amplitude and phase measurement: depending on spacing parameter
Finite-Key Length

The key is ...

- **composable** secure
- provides security against **coherent attacks**

Experimental constraints:

- Alice’s measurements are modeled by projections onto spectrum of quadrature operator for amplitude and phase (parameter: $\delta, \alpha$)
- subsequent measurements commute
- trusted source in Alice’s lab of Gaussian states (can be relaxed)
- No assumptions about Bob’s measurements: **one-sided device independent**
Finite-Key Rates

Key Rate $\ell/N$ depending on symmetric losses for two-mode squeezed state

- input squeezing/antisqueezing 11dB/16dB *
- error correction efficiency of 95%
- excess noise of 1% *
- additional symmetric losses of ...

* T. Eberle et al., arXiv:1110.3977

\[ \epsilon_s = \epsilon_c = 10^{-6} \]

Plot: FF et al., PRL 109, 100502 (2012)
Key Rate versus Losses

Key rate versus losses for $N=10^9$ sifted signal:

$\epsilon_s = \epsilon_c = 10^{-6}$

Plot: FF et al., PRL 109, 100502 (2012)
Security Analysis Based on Uncertainty Relation

Extractable key length:

$$\ell = H^\varepsilon_{\min}(X_A|E)_\omega - \ell_{EC} - O(\log \frac{1}{\epsilon})$$

**Goal:** bound for $H^\varepsilon_{\min}(X_A|E)_\omega$

Key ingredient: **Uncertainty relation with side-information**

Entropic Uncertainty Relation with Side Information

\[ \theta_i = 0 : \text{Amplitude} \]
\[ \theta_i = 1 : \text{Phase} \]

\[ \theta \in_R \{0, 1\}^n \]

Eve

Alice

Bob

\[ X_A \]

\[ \omega_{ABC} \]
Entropic Uncertainty Relation with Side Information

\[ \theta_i = \begin{cases} 0 : \text{Amplitude} \\ 1 : \text{Phase} \end{cases} \]

\[ \theta \in \mathbb{R} \{0, 1\}^n \]

\[ H_{\min}^\epsilon (X_A \mid E \Theta)_\omega \geq \log \frac{1}{c(\delta)} - H_{\max}^\epsilon (X_A \mid \Theta B)_\omega \]

uncertainty Eve has about outcome of Alice

uncertainty of Bob about outcome of Alice

\[ c(\delta) = \left\| Q([0, \delta]) P([0, \delta]) \right\|^2 \approx \frac{\delta^2}{2\pi} \]

complementary of the measurements

Entropic Uncertainty Relation with Side Information

$\theta_i = 0$: Amplitude
$\theta_i = 1$: Phase

$\theta \in \mathbb{R} \{0, 1\}^n$

$X_A$

$H_{\min}^\varepsilon (X_A | E \Theta)_\omega \geq \log \frac{1}{c(\delta)} - H_{\max}^\varepsilon (X_A | \Theta B)_\omega$

$\geq \frac{1}{c(\delta)} - H_{\max}^\varepsilon (X_A | X_B)_\omega$

Data processing inequality

Entropic Uncertainty Relation with Side Information

\[ \theta_i = 0 : \text{Amplitude} \]

\[ \theta_i = 1 : \text{Phase} \]

\[ \theta \in_R \{0, 1\}^n \]

\[ X_A \]

\[ H^\varepsilon_{\text{min}}(X_A | E\Theta)_\omega \geq \log \frac{1}{c(\delta)} - H^\varepsilon_{\text{max}}(X_A | \Theta B)_\omega \]

\[ \geq \frac{1}{c(\delta)} - H^\varepsilon_{\text{max}}(X_A | X_B)_\omega \]

Data processing inequality

Correlation betw. Alice & Bob

Correlation between Alice & Bob

Correlation between Alice and Bob can be bounded in terms of the Hamming distance of a random sample

\[ d(Y_A, Y_B) = \frac{1}{k} \sum_{i=1}^{k} |Y_A^i - Y_B^i| \leq d_0 \]

via

\[ H_{\text{max}}^\varepsilon(X_A | X_B)_\omega \leq n \log \gamma (d(Y_A, Y_B) + \mu) \]

Combining with Uncertainty Relation:

\[ \ell = n \left[ \log \frac{1}{c(\delta)} - \log \gamma (d_0 + \mu) \right] - O(\log \frac{1}{\epsilon}) - \ell_{\text{EC}} \]
Conclusion

Advantage:

- one-sided device independent (e.g. local oscillator included)
- direct approach (no additional measurements compared to post-selection approach)
- no state tomography
- robust under small deviations of experimental parameters

Problems:

- very sensitive to noise
- asymptotically not optimal: Uncertainty relation not tight for the Gaussian states used in the protocol

Implementation in Leibniz University in Hannover:

Crypto on Campus: T. Eberle, V. Händchen, J. Duhme, T. Franz, R. F. Werner, and R. Schnabel
Thank you for your attention!